Static and Dynamic Moduli

ROSE

Rock Physics and Geomechanics

Course 2012

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What do we mean with "static" and "dynamic" moduli?

Elastic wave velocities

$$V_P = \sqrt{\frac{\lambda_e + 2G_e}{\rho}}$$

$$V_S = \sqrt{\frac{G_e}{\rho}}$$

⇨

$$G_e = \rho V_s^2$$

$$\lambda_e = \rho V_p^2 - 2\rho V_s^2$$

Dynamic moduli

Stress and strain measured in a rock mechanical test

$$E = \frac{\Delta\sigma_z}{\Delta\varepsilon_z}$$

Static moduli

In general:

$$E_{stat} \neq E_{dyn}$$

$$E_{stat} < E_{dyn}$$

Note: $E_{stat} : E_{dyn}$ is not a constant ratio – it changes with stress!

In saturated rocks,

- Static deformation is often <u>drained</u>
- Dynamic moduli are always <u>undrained</u>

Occasionally used definition:

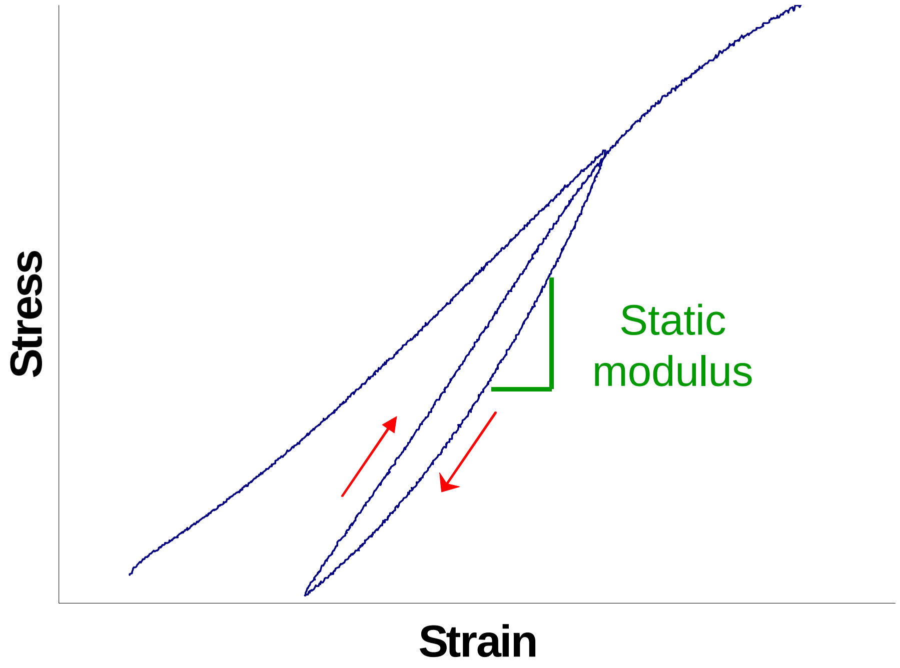
"Static modulus = drained modulus"

"Dynamic modulus" = undrained modulus"

Not recommended!

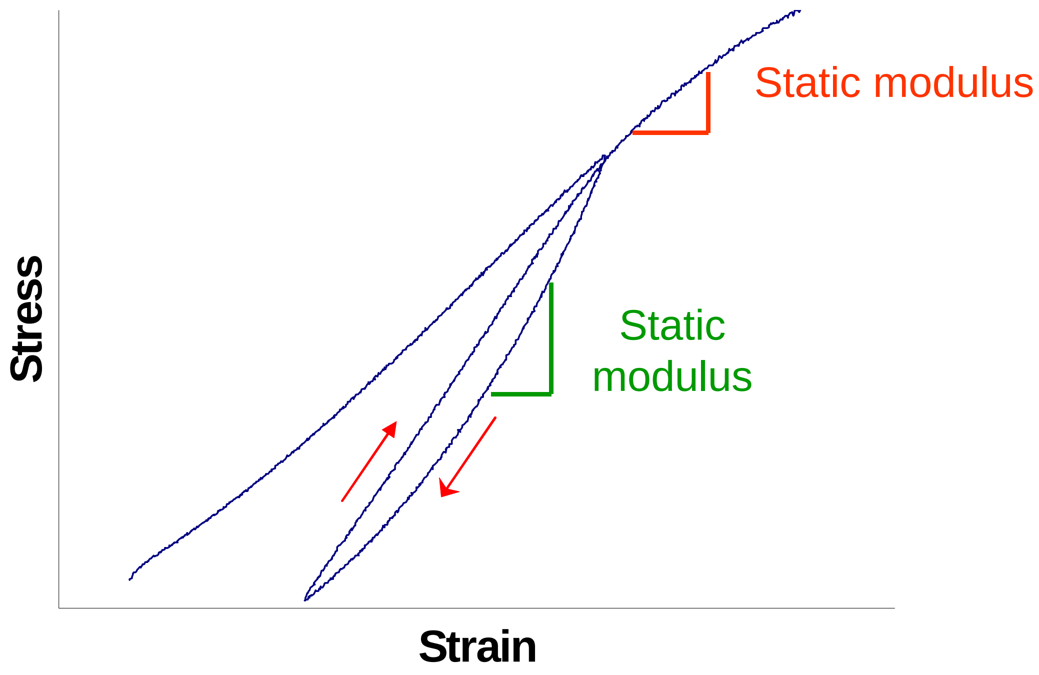
Alternative definition, also used:

"Static modulus" = slope of stress-strain curve measured during unloading

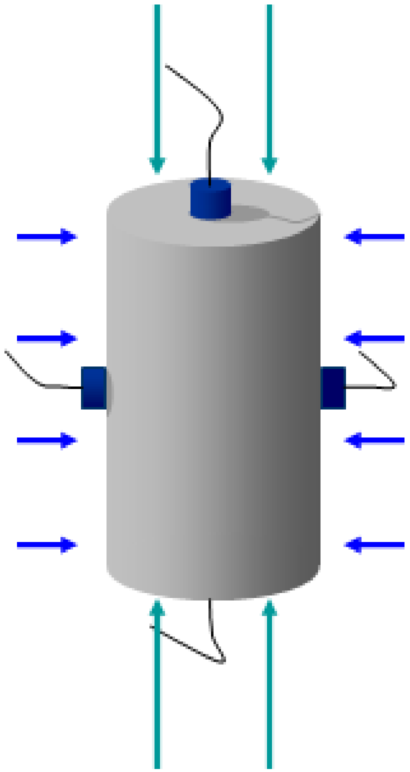


Our definition:

"Static modulus" = slope of stress-strain curve



Laboratory tests:



Standard triaxial set-up + acoustics

Measurements:

Stress

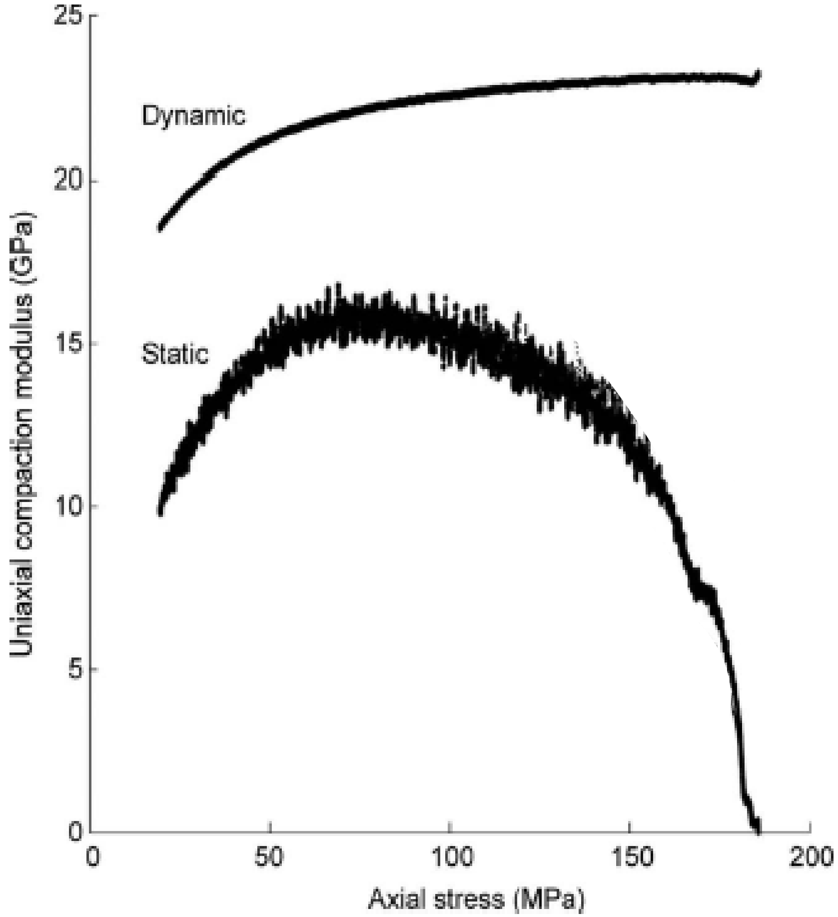
Strain

Acoustic wave velocities

Enables simultaneous measurements of
- static moduli (slope of stress-strain curve)
- dynamic moduli (density x velocity^2)

Static and dynamic moduli of soft rocks are different.

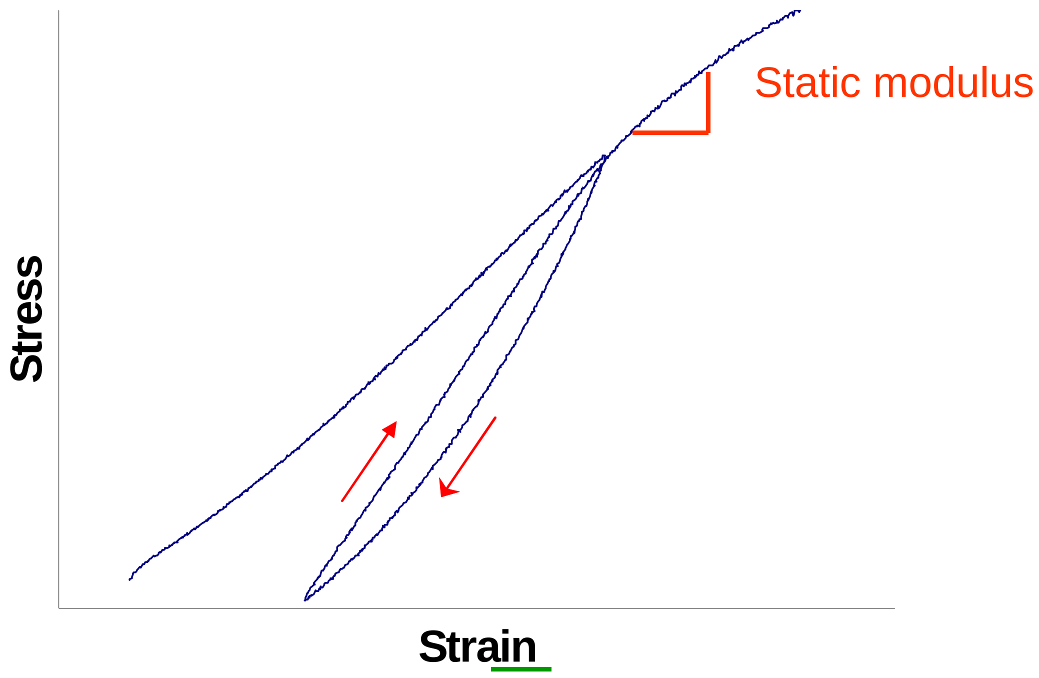
The difference changes along the stress path.



Potential causes for the difference between static and dynamic moduli:

- Strain rate
- Length of stress path
- Stress history
- Rock volume involved
- Drainage conditions
- Anisotropy

First: consider the static modulus measured during **initial loading**



Dry rock

Fjær (1999):

We introduce **a parameter P**, defined as**:**

$$P = \frac{\Delta\varepsilon_v - \Delta\varepsilon_{v,e}}{3\Delta\sigma}$$

$\Delta\varepsilon_v$ - total volumetric strain

$$\Delta\varepsilon_{v,e} \equiv \frac{\Delta\sigma}{K_e}$$ - elastic strain

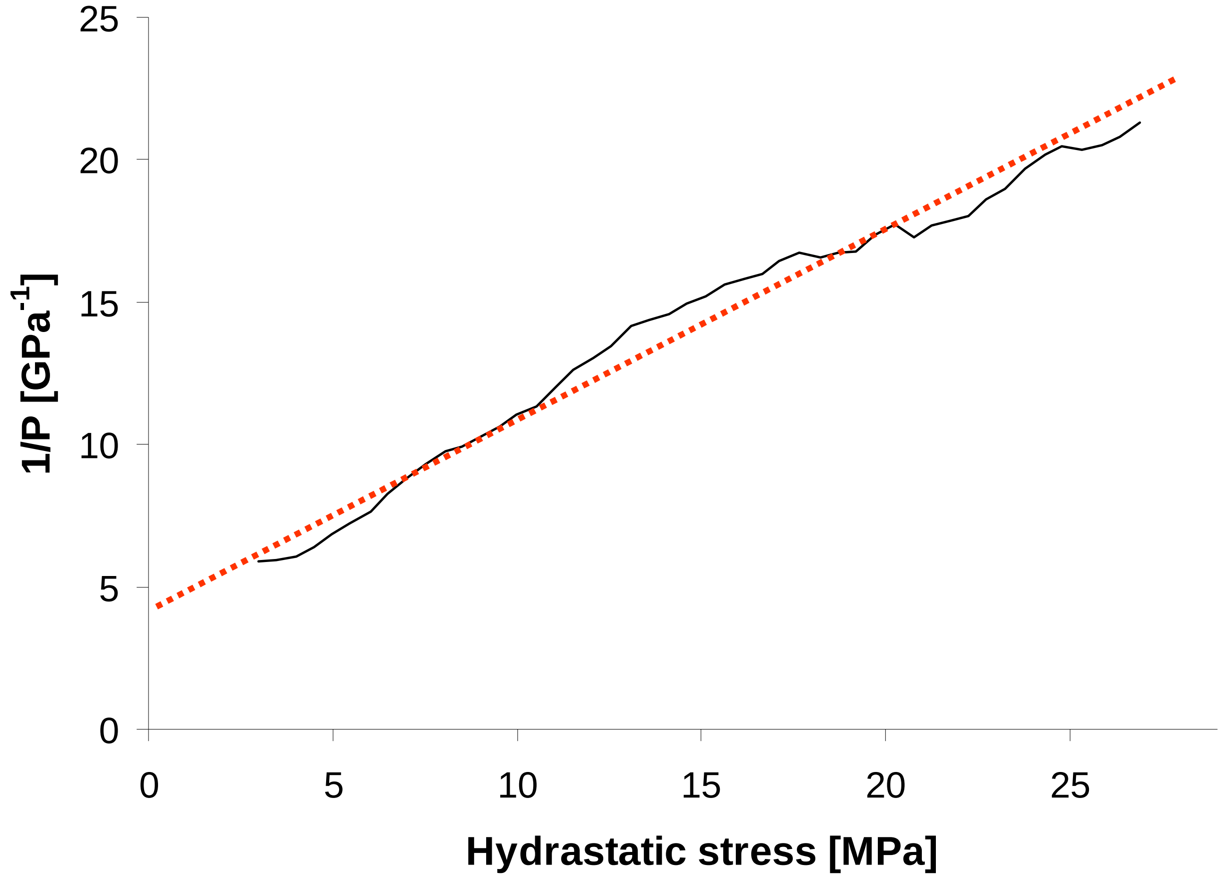
P is a measure of the inelastic part of the deformation caused by a compressive hydrostatic stress increment.

$\Rightarrow$

$$K = \frac{K_e}{1+3PK_e}$$

K = Static bulk modulus
K_e = Dynamic bulk modulus

Observations



$$P = \frac{\varepsilon_g}{\sigma + T}$$

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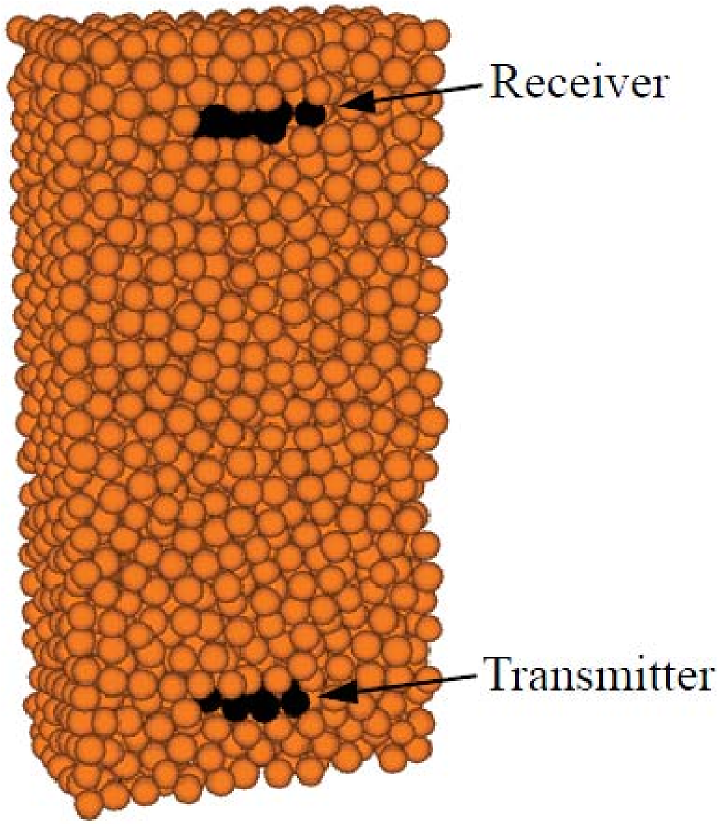
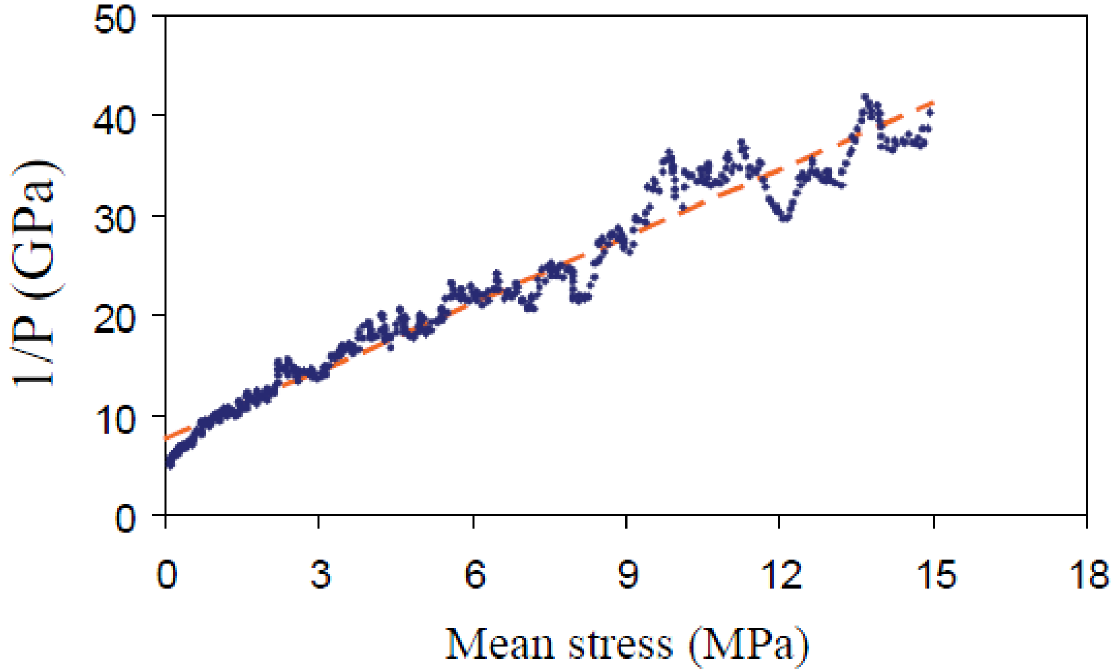
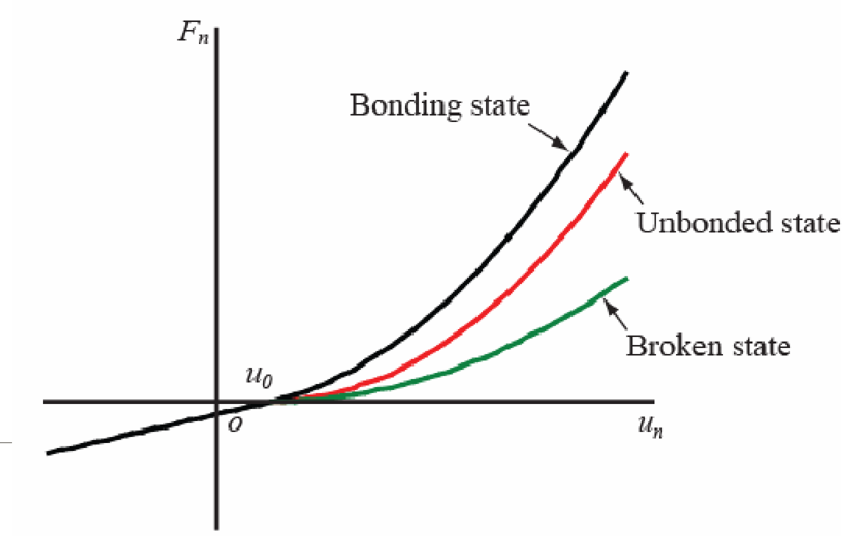
PFC^{3D}-simulation

Li & Fjær, 2008







Dry rock

Fjær (1999):

We introduce **a parameter** $\boldsymbol{F}$, defined as**:**

$$F = \frac{\Delta\varepsilon_z - \Delta\varepsilon_{z,e} - \Delta\varepsilon_{z,p}}{\Delta\varepsilon_z}$$

F is a measure of the inelastic part of the deformation caused by a shear stress increment.

$\Delta\varepsilon_z$ - total axial strain

$$\Delta\varepsilon_{z,e} \equiv \frac{\Delta\sigma}{E_e}$$ - elastic strain

$$\Delta\varepsilon_{z,p} = P_z \Delta\sigma_z$$

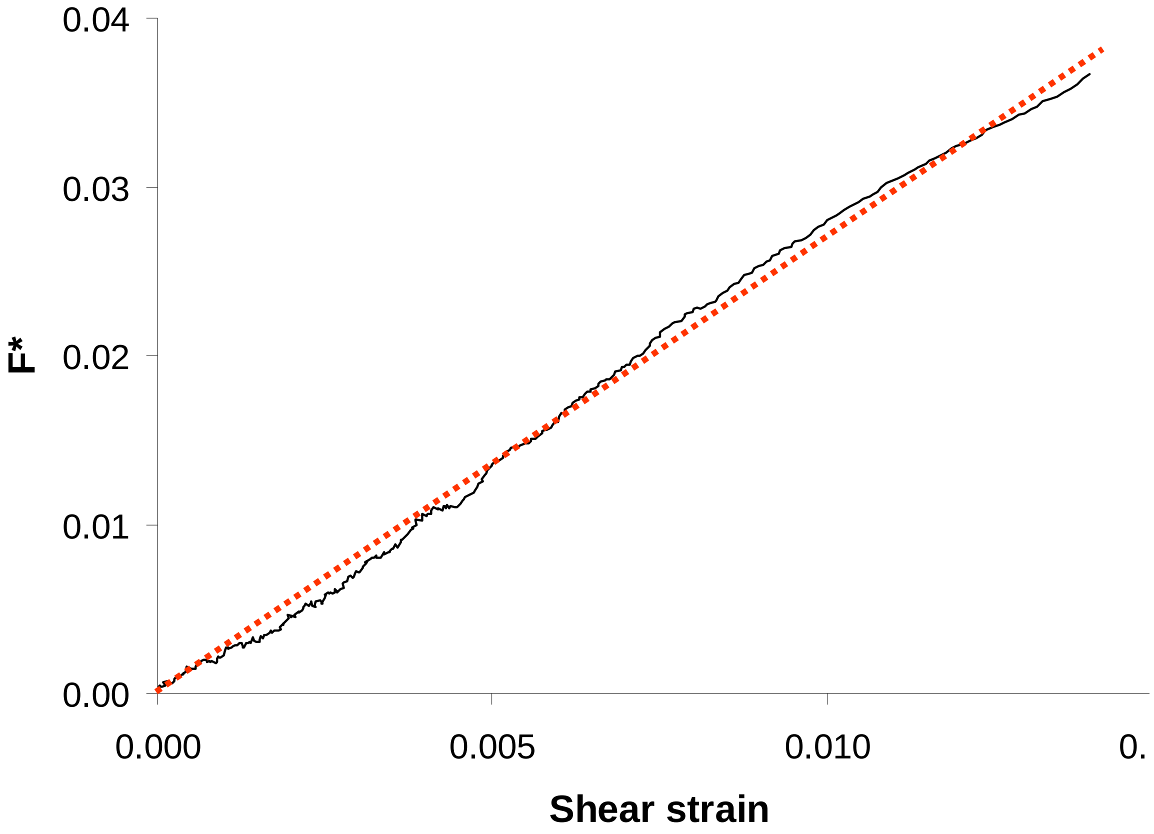
$\Rightarrow$

$$E = \frac{E_e}{1 + P_z E_e}(1 - F)$$

E = Static Young's modulus
E_e = Dynamic Young's modulus

Observations

$$F^* = F\sqrt{\sigma_z + \sigma_r + S}$$



$$F = A\frac{\varepsilon_z - \varepsilon_r - \varepsilon_o}{\sqrt{\sigma_z + \sigma_r + S}}$$

Discussion: the F - parameter

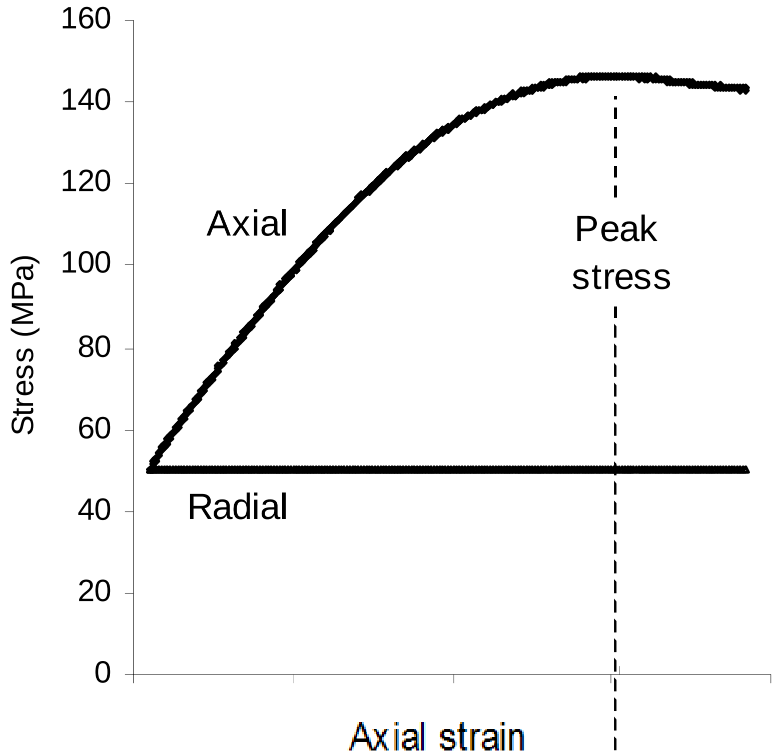
$$E = \frac{E_e}{1 + P_z E_e}(1 - F)$$

Note:

Since $E \propto (1 - F)$

$\Rightarrow$ when $F = 1$ then $E = 0$

$\Rightarrow$ peak stress



$F = 1 \leftrightarrow$ rock strength

We have a set of equations……

These represent a constitutive model for the rock

We may use it to predict rock behavior, and thereby derive mechanical properties for the rock

$$K = \frac{K_e}{1 + 3PK_e}$$

$$E = \frac{E_e}{1 + P_z E_e}\left(1 - F\right)$$

$$P = \frac{\varepsilon_g}{\sigma + T}$$

$$F = A\frac{\varepsilon_z - \varepsilon_r - \varepsilon_o}{\sqrt{\sigma_z + \sigma_r + S}}$$

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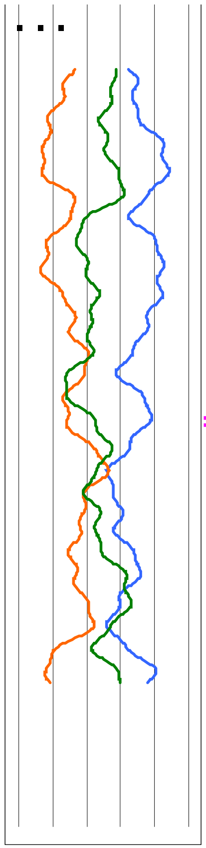
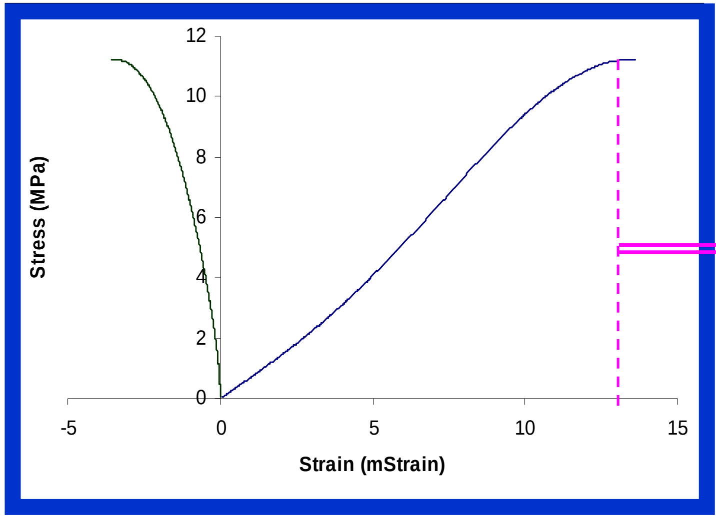
Application for logging purposes

Porosity, Density, Sonic, ...



Provides calibration for the model

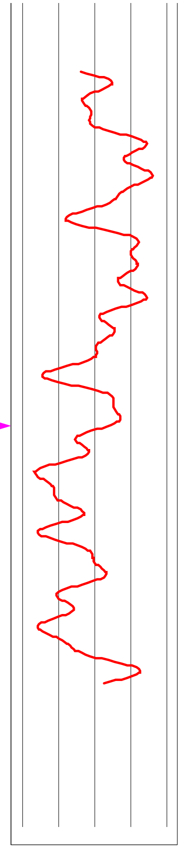
Constitutive model



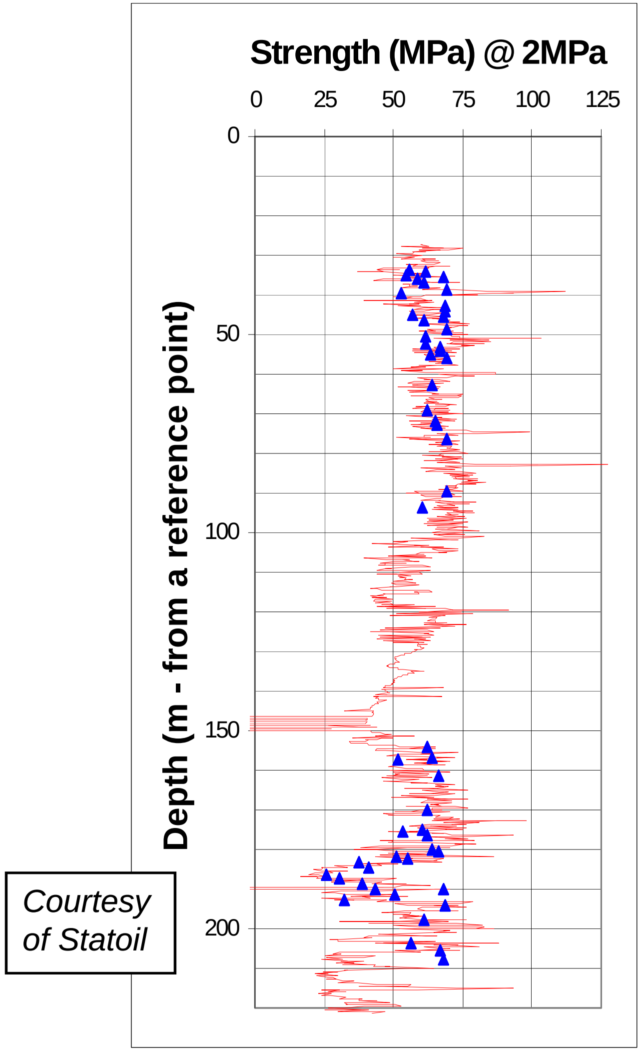
Simulates rock mechanical test on fictitious core

Provides strength and stiffness

Strength



… an example:



Prediction from logs

Core measurements

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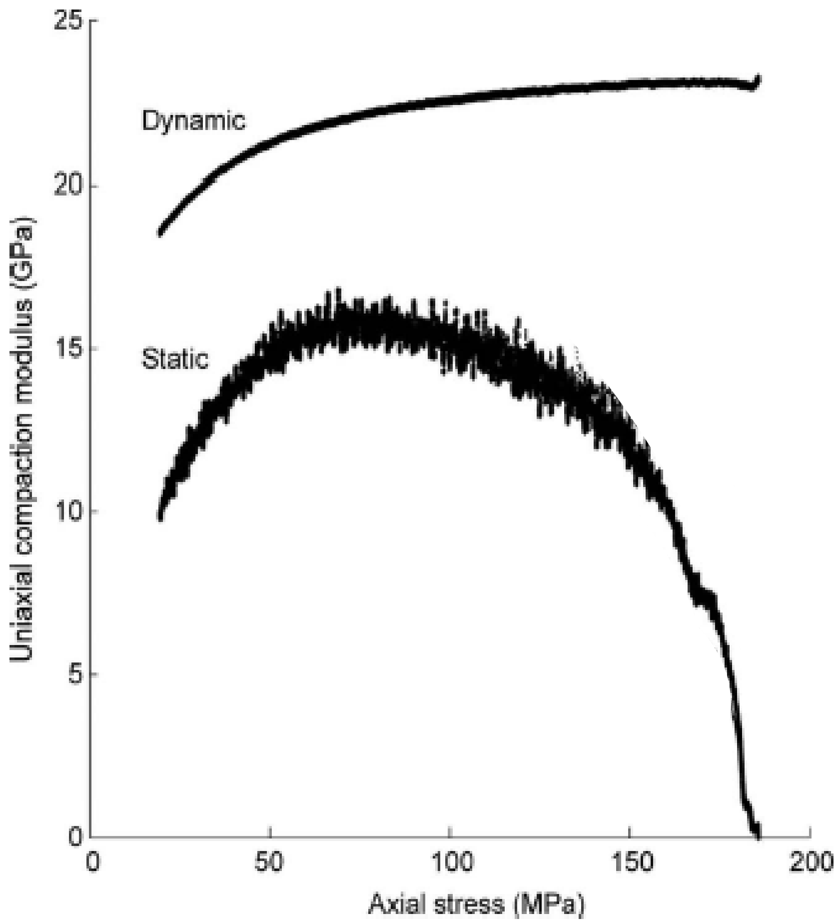
Static and dynamic moduli of soft rocks are different.

The difference changes
along the stress path.



Potential causes for the difference between static and dynamic moduli:

- Strain rate
- Length of stress path
- Stress history
- Rock volume involved
- Drainage conditions
- Anisotropy

1. Stress path:

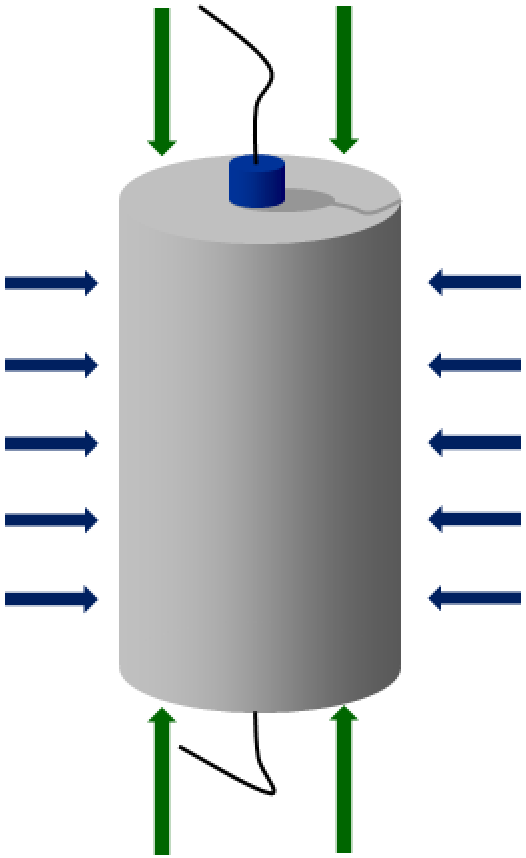
Uniaxial strain (K_0)

Static modulus = slope of stress-strain curve:

$$H = \frac{\Delta \sigma_z}{\Delta \varepsilon_z}$$

Dynamic modulus given by axial P-wave velocity:

$$H_e = \rho V_P^2$$



Potential causes for the difference between static and dynamic moduli:

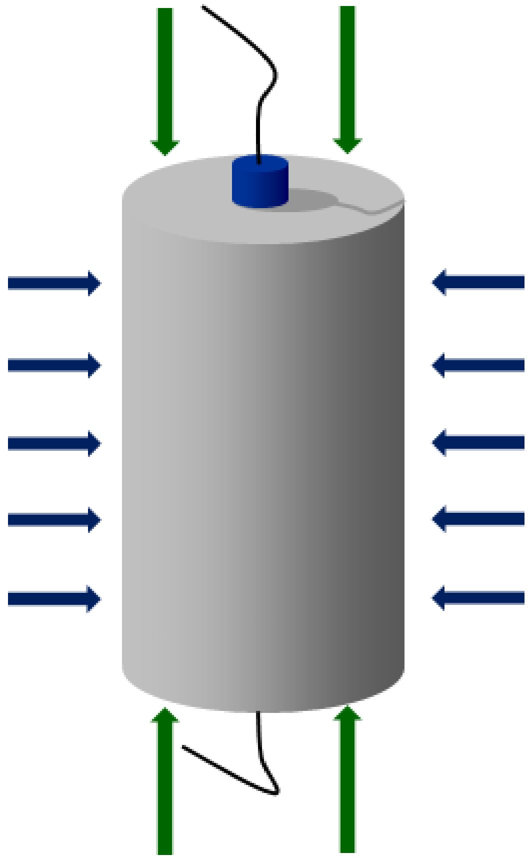
- Strain rate
- Length of stress path
- Stress history
- ~~Rock volume involved~~
- Drainage conditions
- ~~Anisotropy~~

2. Saturation:

Static modulus: usually drained

Dynamic modulus: undrained

Irrelevant for dry rocks



Potential causes for the difference between static and dynamic moduli:

- Strain rate
- Length of stress path
- Stress history
- ~~Rock volume involved~~
- ~~Drainage conditions~~
- ~~Anisotropy~~

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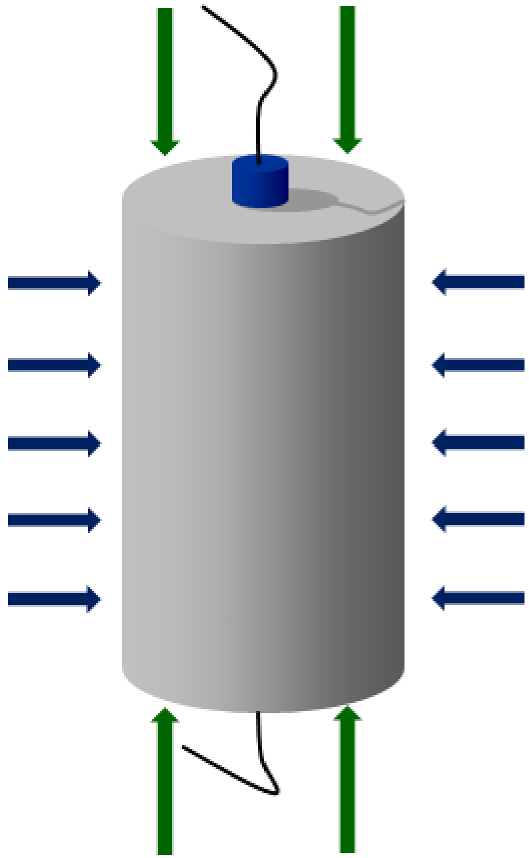
3. Loading direction:

Friction controlled shear sliding
of closed cracks
- the only non-elastic process
active during unloading



Potential causes for the difference between static and dynamic moduli:

- Strain rate
- Length of stress path
- ~~Stress history~~
- ~~Rock volume involved~~
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- ~~Anisotropy~~

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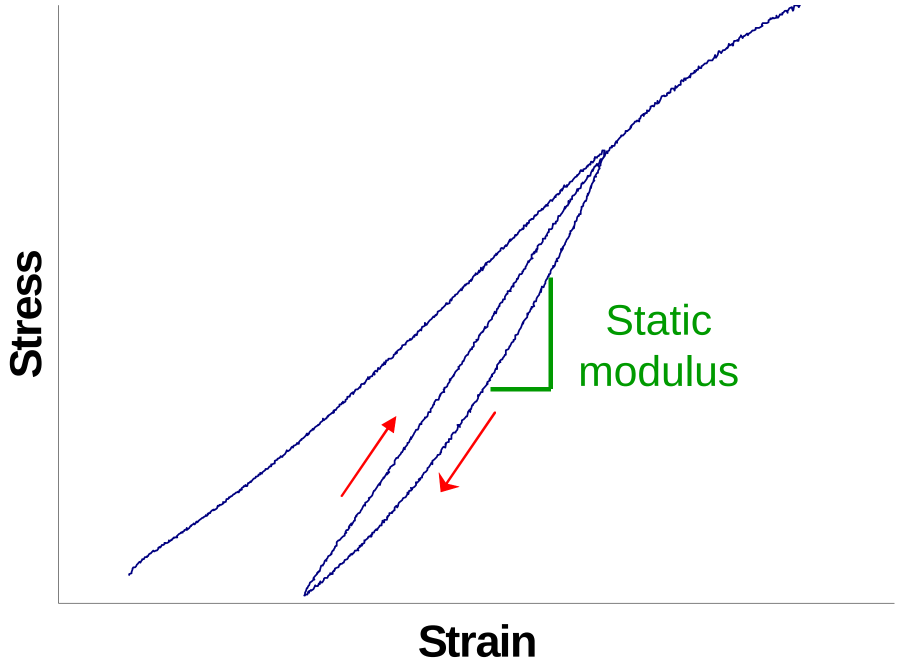
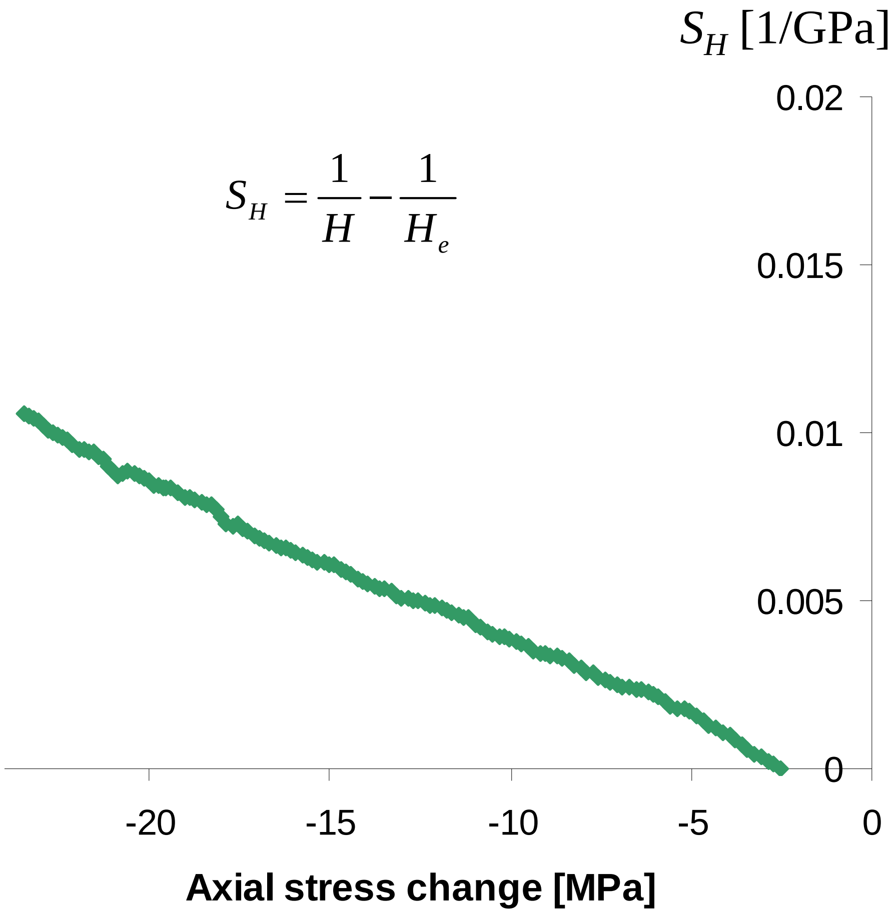
Fjær et al. (2011):

Consider the slope of the stress-strain curve during unloading





$$S_H = \frac{1}{H} - \frac{1}{H_e}$$

Observation:

Non-elastic compliance $S_H = \frac{1}{H} - \frac{1}{H_e}$ increases linearly with decreasing stress

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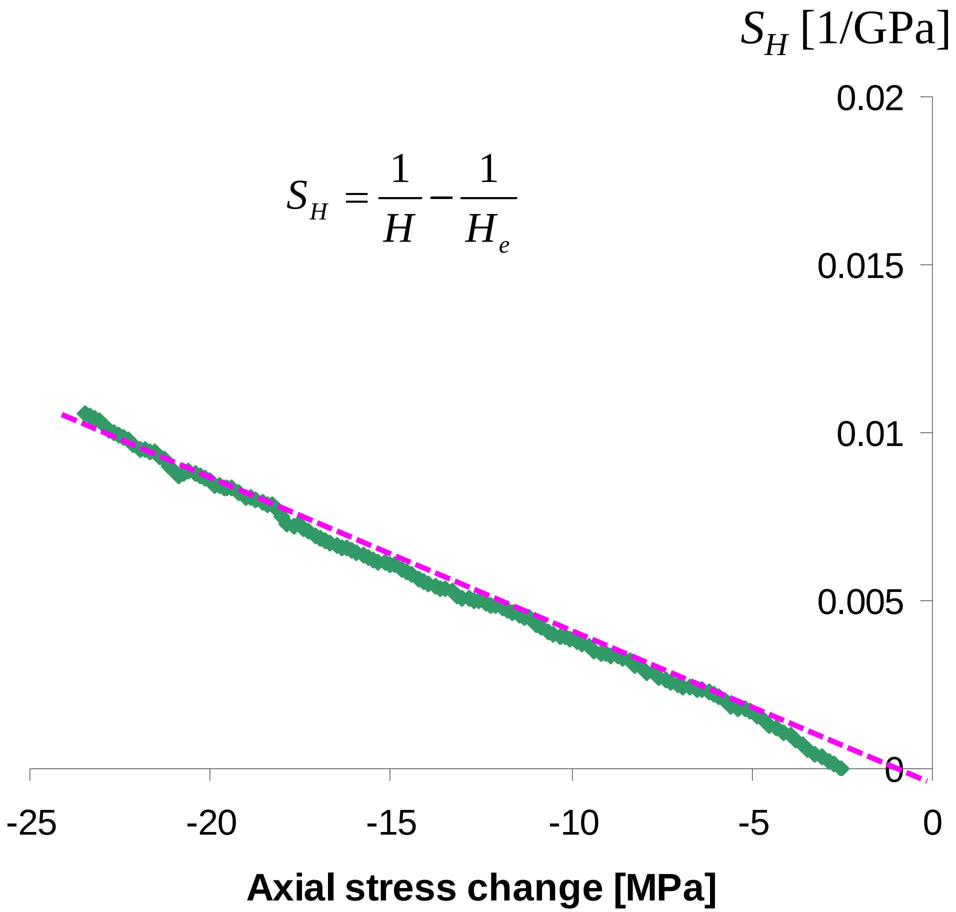
Assumption:
Linear extrapolation towards the beginning of the unloading path provides estimate of behavior for vanishing length of stress path



$$S_H = \frac{1}{H} - \frac{1}{H_e}$$

Observation:

Non-elastic compliance $S_H = \frac{1}{H} - \frac{1}{H_e}$ increases linearly with decreasing stress

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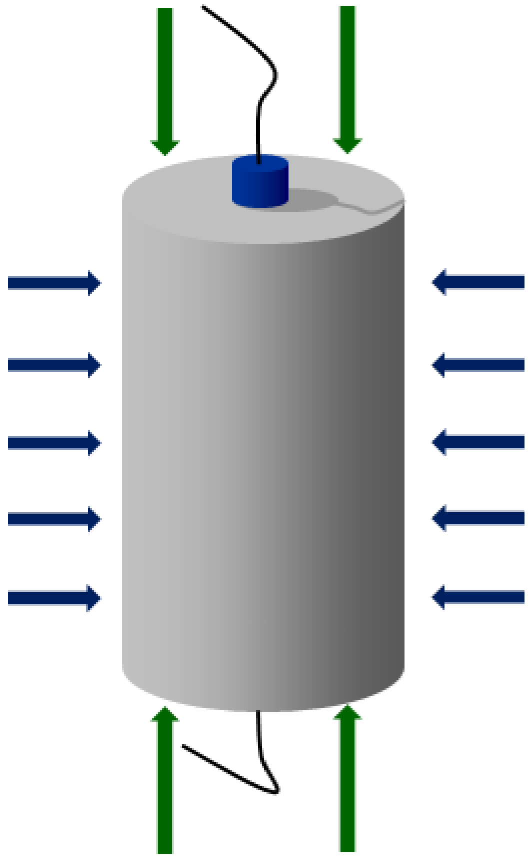
4. Loading direction:

Linear extrapolation towards the beginning of the unloading path provides estimate of behavior for vanishing length of stress path



Potential causes for the difference between static and dynamic moduli:

- Strain rate
- ~~Length of stress path~~
- ~~Stress history~~
- ~~Rock volume involved~~
- ~~Drainage conditions~~
- ~~Anisotropy~~

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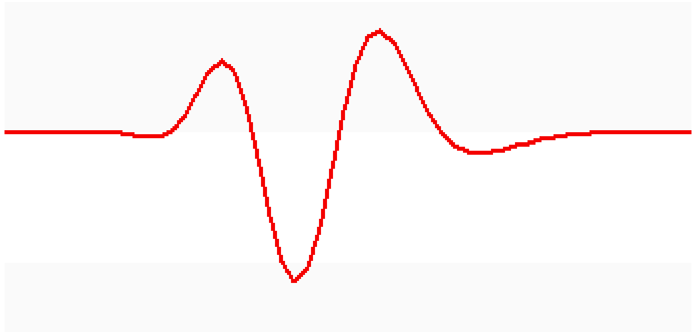
Strain rate



Average strain rate for dynamic measurements:

$$\langle \dot{\varepsilon} \rangle = 4 f \Delta\varepsilon$$

$\Delta\varepsilon \sim 10^{-7}$ = strain amplitude

$f = 5 \cdot 10^{5}$ Hz = frequency

$$\Rightarrow \langle \dot{\varepsilon} \rangle \approx 10^{-1}\ \mathrm{s}^{-1}$$

For "static" deformations

$$\dot{\varepsilon} \approx 10^{-6}\ \mathrm{s}^{-1}$$

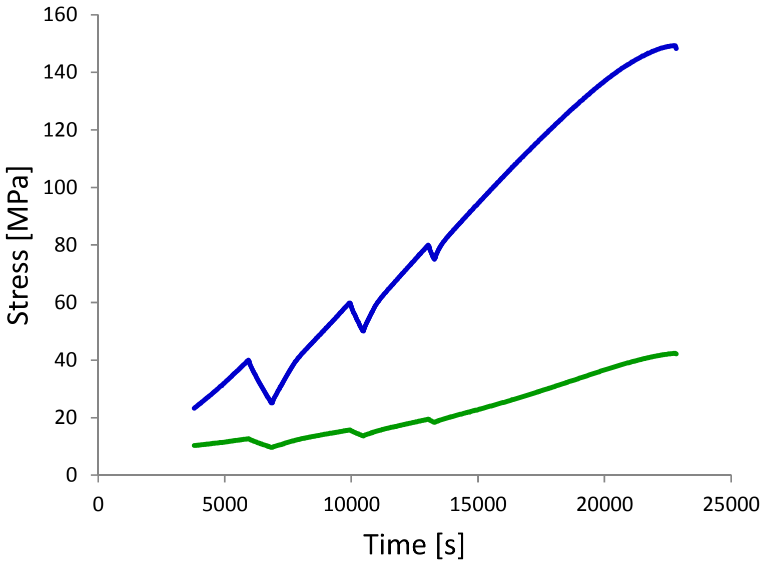
Corresponds to an acoustic wave with $f \approx 1$ Hz

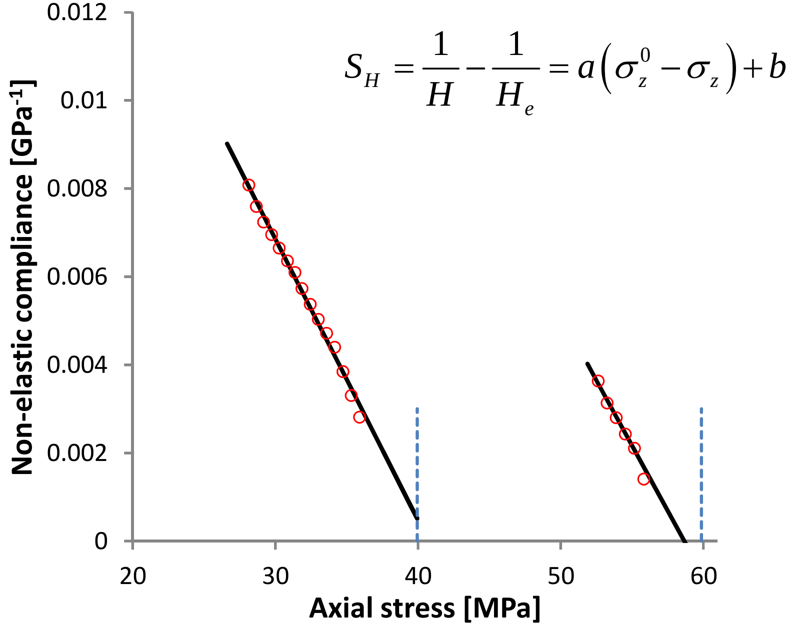
If strain rate is the only cause for the difference between the static and dynamic moduli, then

static modulus $\leftrightarrow$ dynamic modulus at $\approx$ 1 Hz

Castlegate sandstone

Dry, clay free – presumably no significant dispersion





- K0 stress path
- Unloading sequences

σ_z MPa	σ_r MPa	b 10^{-4} GPa^{-1}	$V_{Ph} - V_{Pl}$ m/s	V_{Ph} m/s
40	13	5.2 ± 1.8	16 ± 36	3222
60	16	-7.2 ± 2.0	-24 ± 38	3285

Uncertainties: Δb, $\Delta\rho = 1\%$, $\Delta V_{Ph} = 1\%$

No measurable dispersion

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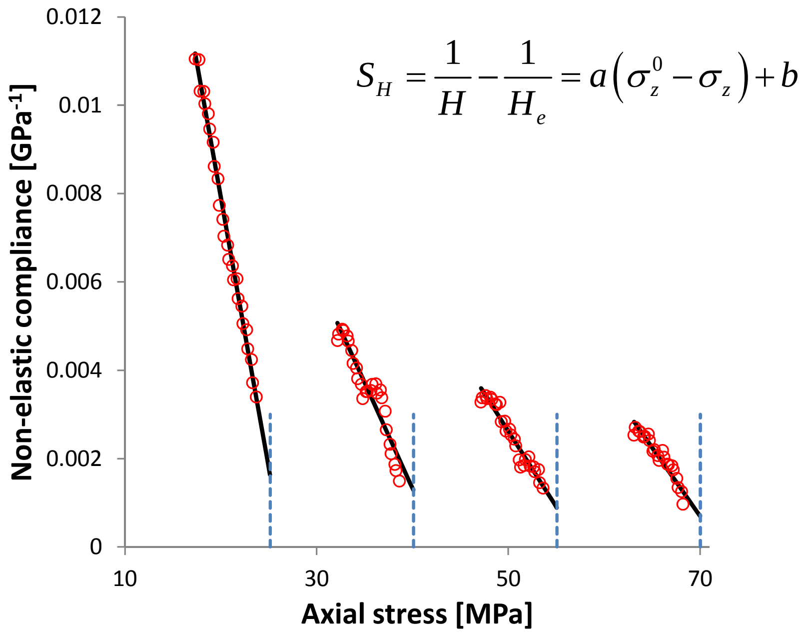
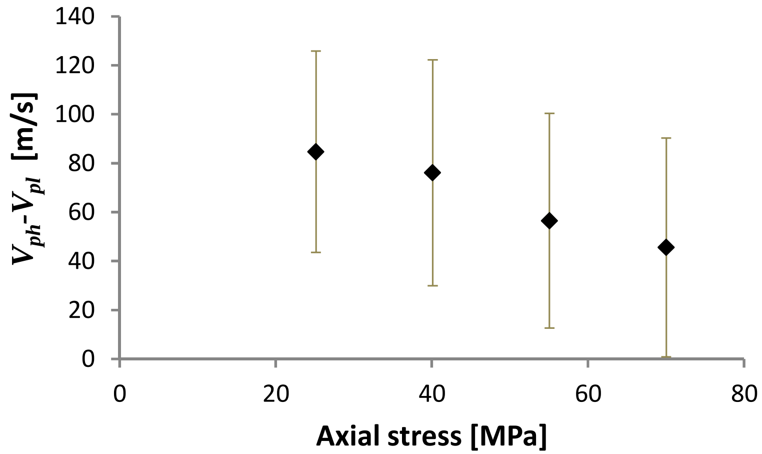
Fjær et al. (2012):

Berea sandstone

Dry, 8% clay – possibly some dispersion





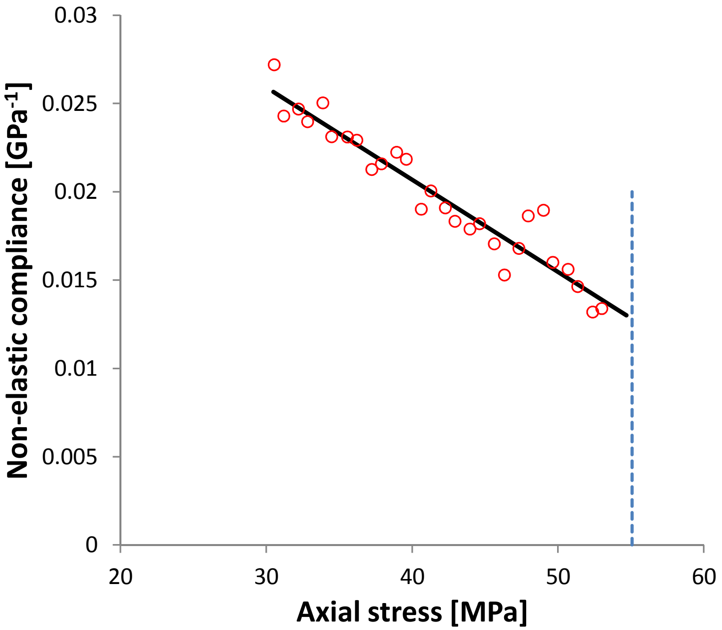
Uncertainties: Δb, $\Delta\rho = 1\%$, $\Delta V_{Ph} = 1\%$

σ_z MPa	σ_r MPa	b 10^{-4} GPa^{-1}	$V_{Ph} - V_{Pl}$ m/s	V_{Ph} m/s
25	11	16.1 ± 2.8	85 ± 41	3703
40	13	12.8 ± 3.6	76 ± 46	3853
55	15	8.8 ± 1.9	56 ± 44	3929
70	17	6.9 ± 2.1	46 ± 45	3963

Significant, measurable dispersion, decreasing with increasing stress

Mancos shale

Saturated, with 13% illite/smectite, 5% kaolinite, etc.
– probably significant dispersion



σ_z MPa	σ_r MPa	b 10^{-4} GPa^{-1}	$V_{Ph} - V_{Pl}$ m/s	V_{Ph} m/s
55	18	129 ± 12	846 ± 112	4163

Uncertainties: Δb, $\Delta\rho = 1\%$, $\Delta V_{Ph} = 1\%$

Significant dispersion,
far beyond the resolution limit
for the method

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Other applications

We measure,

- the axial P-wave velocity $V_{Pz} = \sqrt{\dfrac{C_{33}^e}{\rho}}$

- the axial S-wave velocity $V_{Szr} = \sqrt{\dfrac{C_{44}^e}{\rho}}$

- the axial stress σ_z

- the radial stress σ_r

$$r_{44}^e \equiv \frac{C_{44}^e}{C_{33}^e} = \left(\frac{V_{S,zr}}{V_{P,z}}\right)^2$$

$$r_{13} \equiv \frac{C_{13}}{C_{33}} = \frac{\Delta\sigma_r}{\Delta\sigma_z}$$

We want

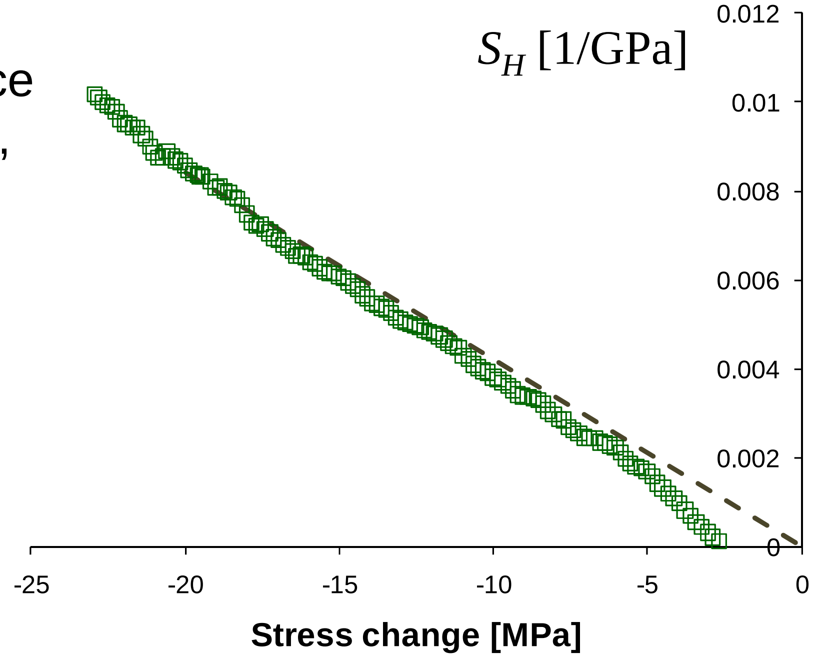
Thomsen's δ $\quad \delta = \dfrac{\left(C_{13}^e + C_{44}^e\right)^2 - \left(C_{33}^e - C_{44}^e\right)^2}{2C_{33}\left(C_{33}^e - C_{44}^e\right)}$

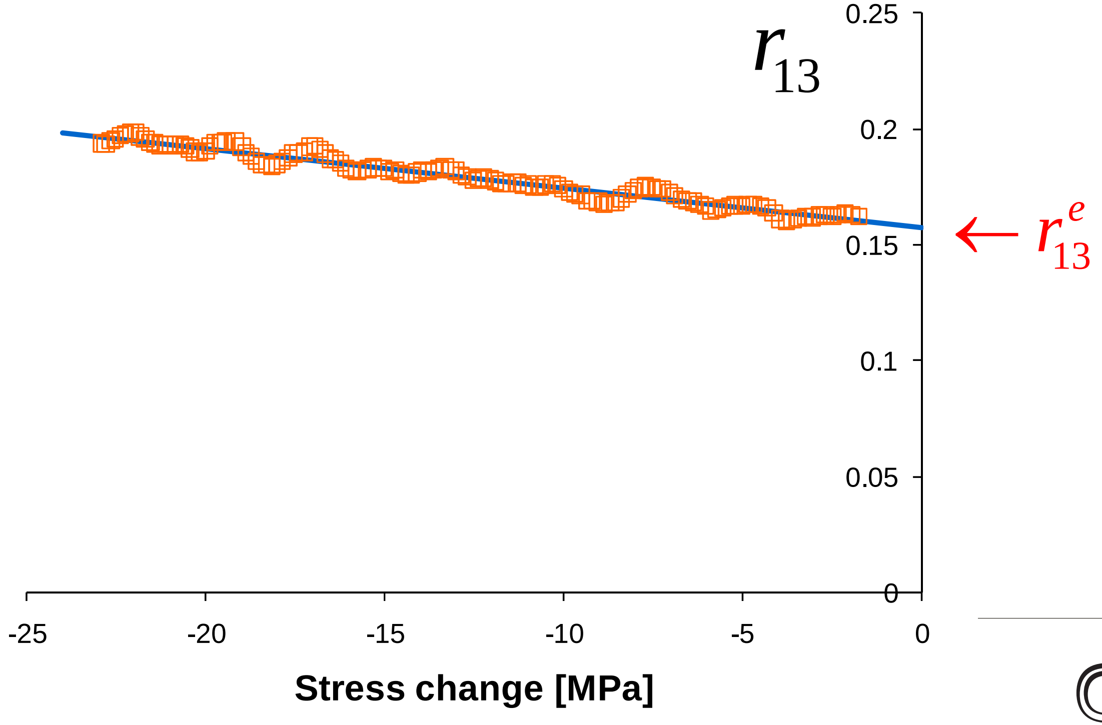
$$\delta = \frac{\left(r_{13}^e + r_{44}^e\right)^2 - \left(1 - r_{44}^e\right)^2}{2\left(1 - r_{44}^e\right)}$$

$\Rightarrow$ We have r_{44}^e & r_{13} , we need r_{13}^e

We have seen that the non-elastic compliance vanish at the turning point on the stress path, (the approach is linear during unloading)

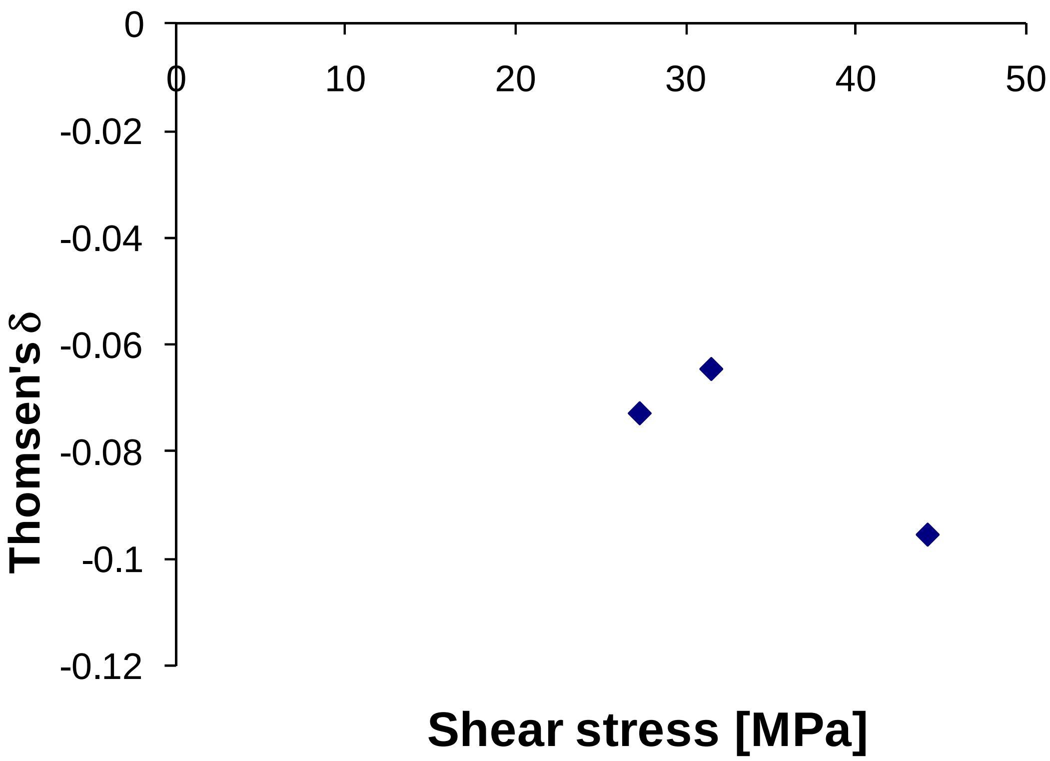
- which means:
static modulus → dynamic modulus at the turning point





Therefore –
We <u>assume</u> that static r_{13} → dynamic r_{13} at the turning point

Example (dry Castlegate sandstone):



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