Crack models

ROSE

Rock Physics and Geomechanics

Course 2012

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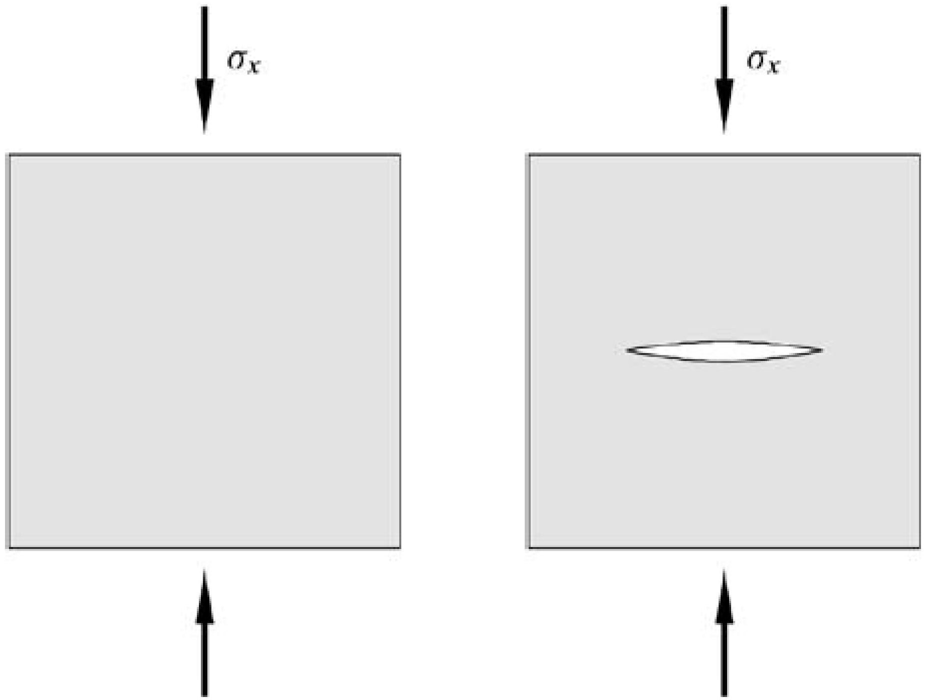
SINTEF NTNU

Cracks have a strong impact on rock behavior

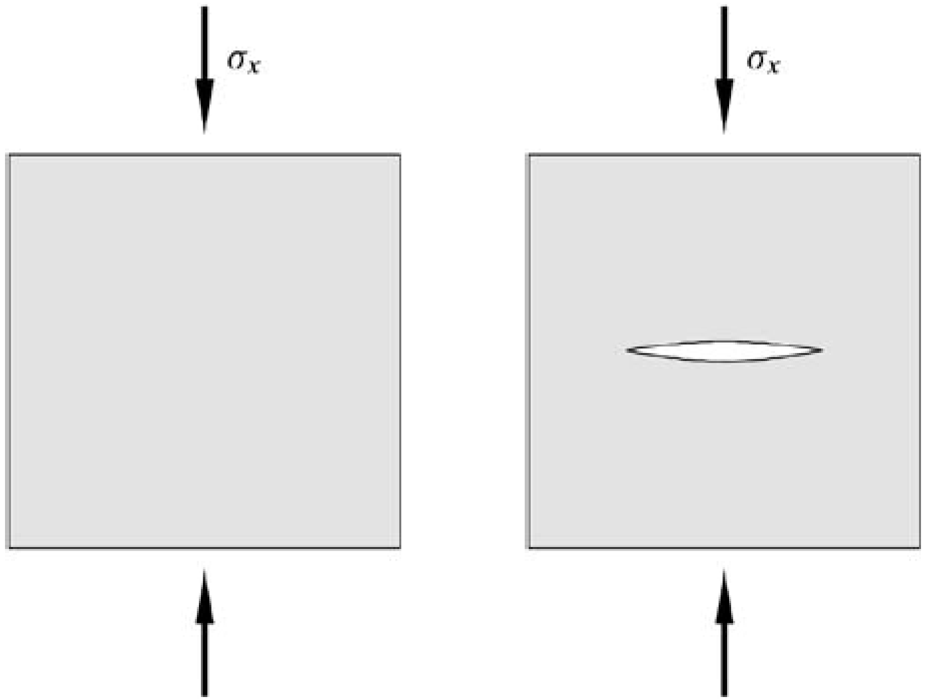


The presence of cracks reduces the stiffness of the rock

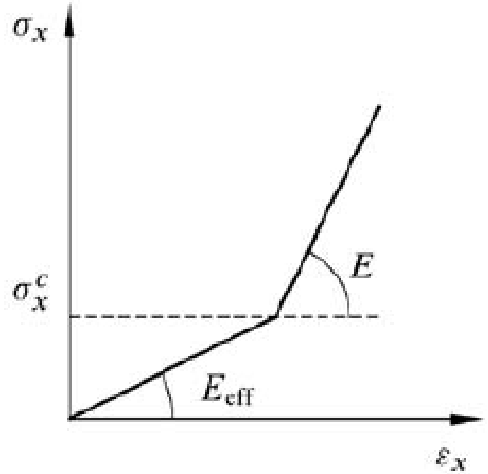
$$E = \frac{\Delta\sigma_x}{\Delta\varepsilon_x}$$

$$E_{\text{eff}} = \frac{\Delta\sigma_x}{\Delta\varepsilon_x} = E\left(1 - Q\xi\right)$$

Cracks have a strong impact on rock behavior

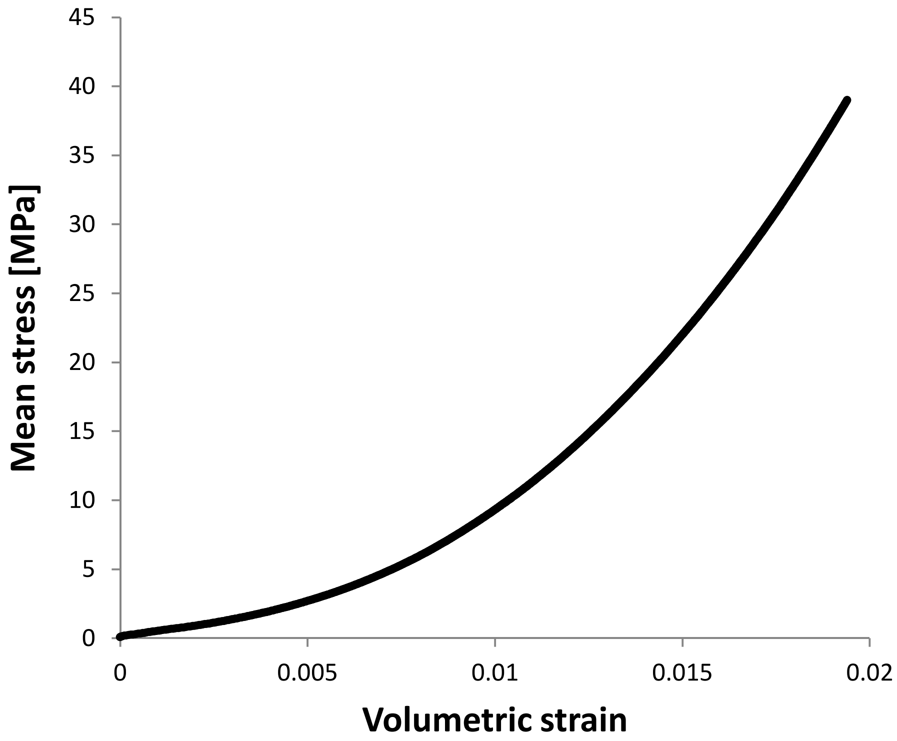


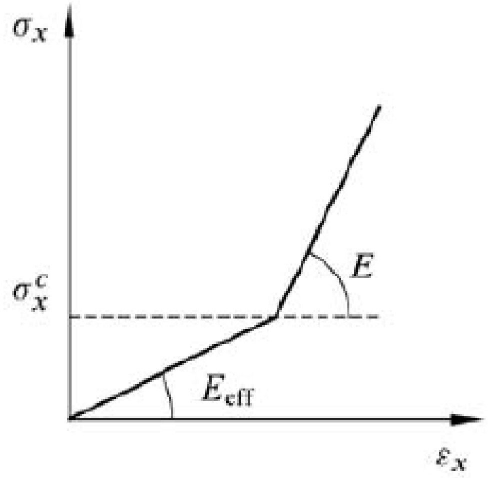
$$E = \frac{\Delta\sigma_x}{\Delta\varepsilon_x} \qquad E_{\text{eff}} = \frac{\Delta\sigma_x}{\Delta\varepsilon_x} = E\left(1 - Q\xi\right)$$



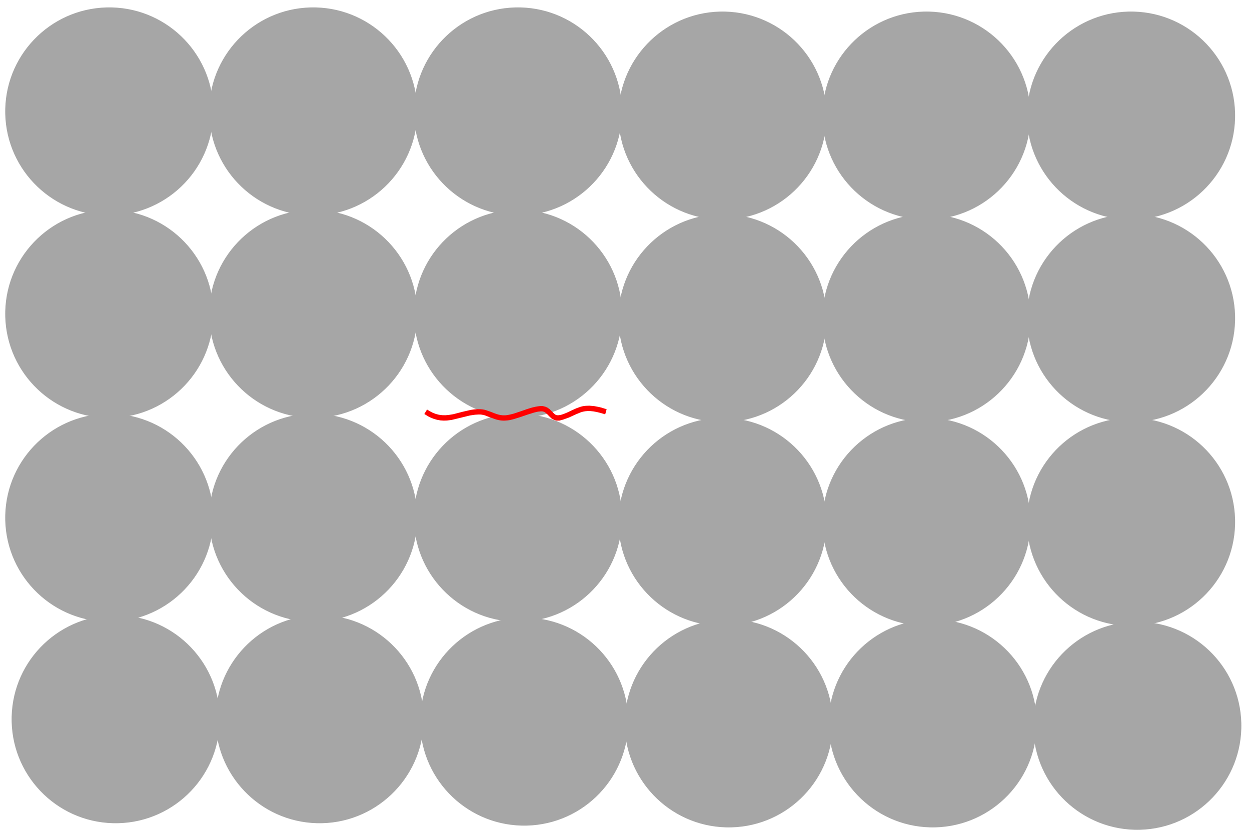
Increasing stress
⇒ Crack closure
⇒ Increasing stiffness

Cracks have a strong impact on rock behavior

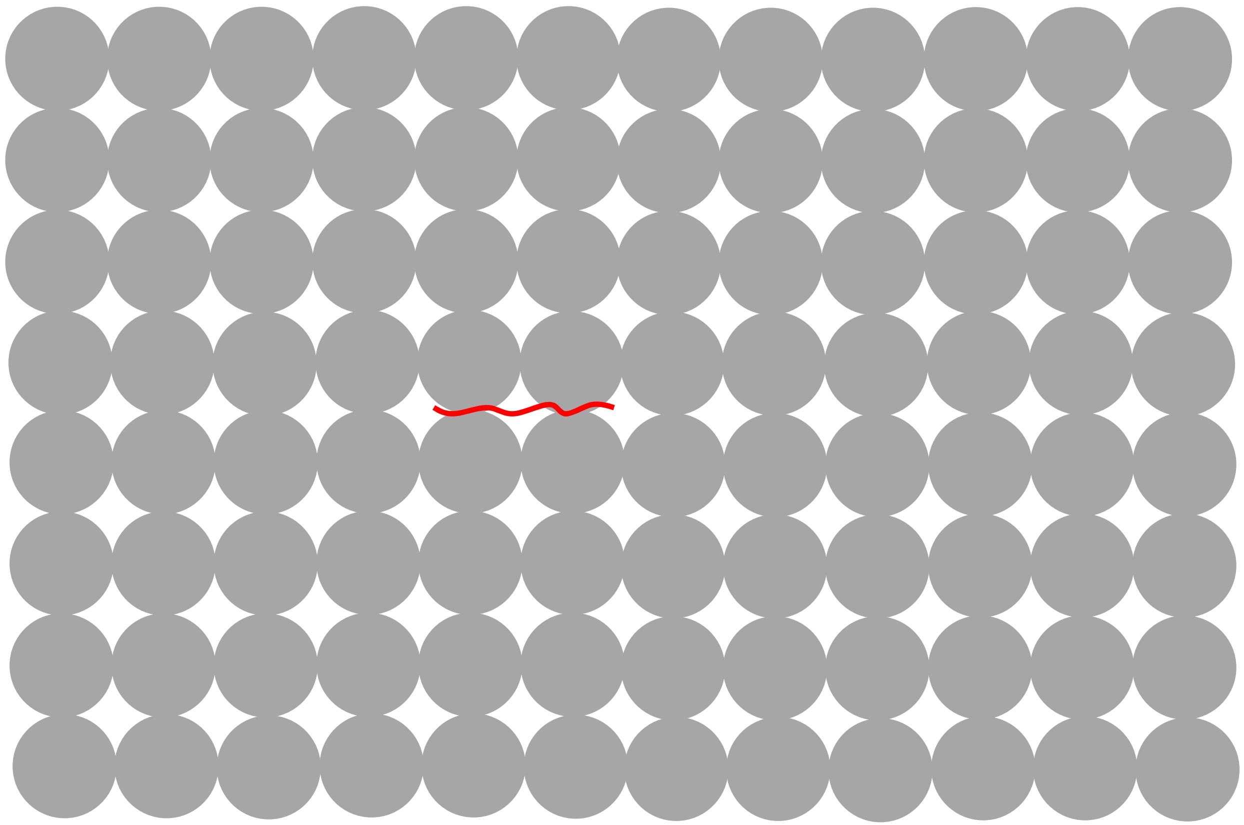




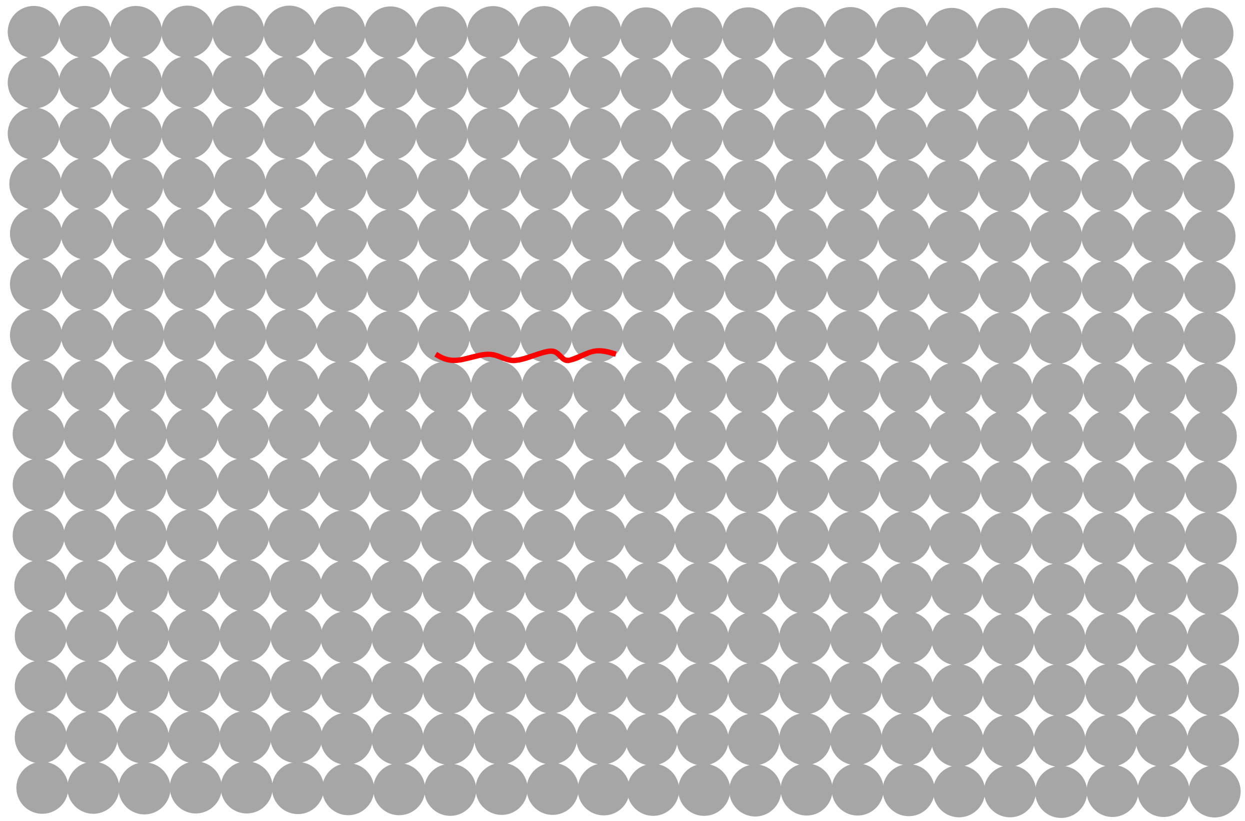
Increasing stress
$\Rightarrow$ Crack closure
$\Rightarrow$ Increasing stiffness



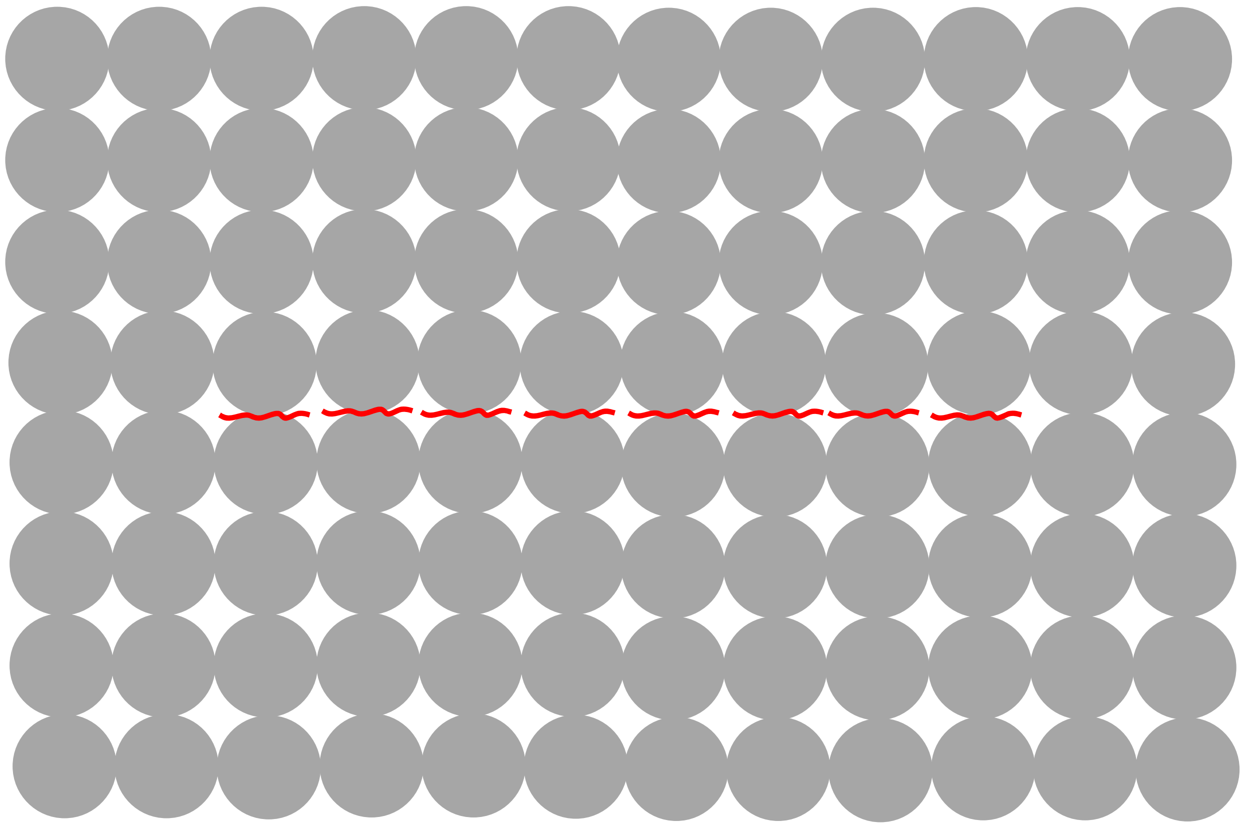
Crack – or failed grain contact?



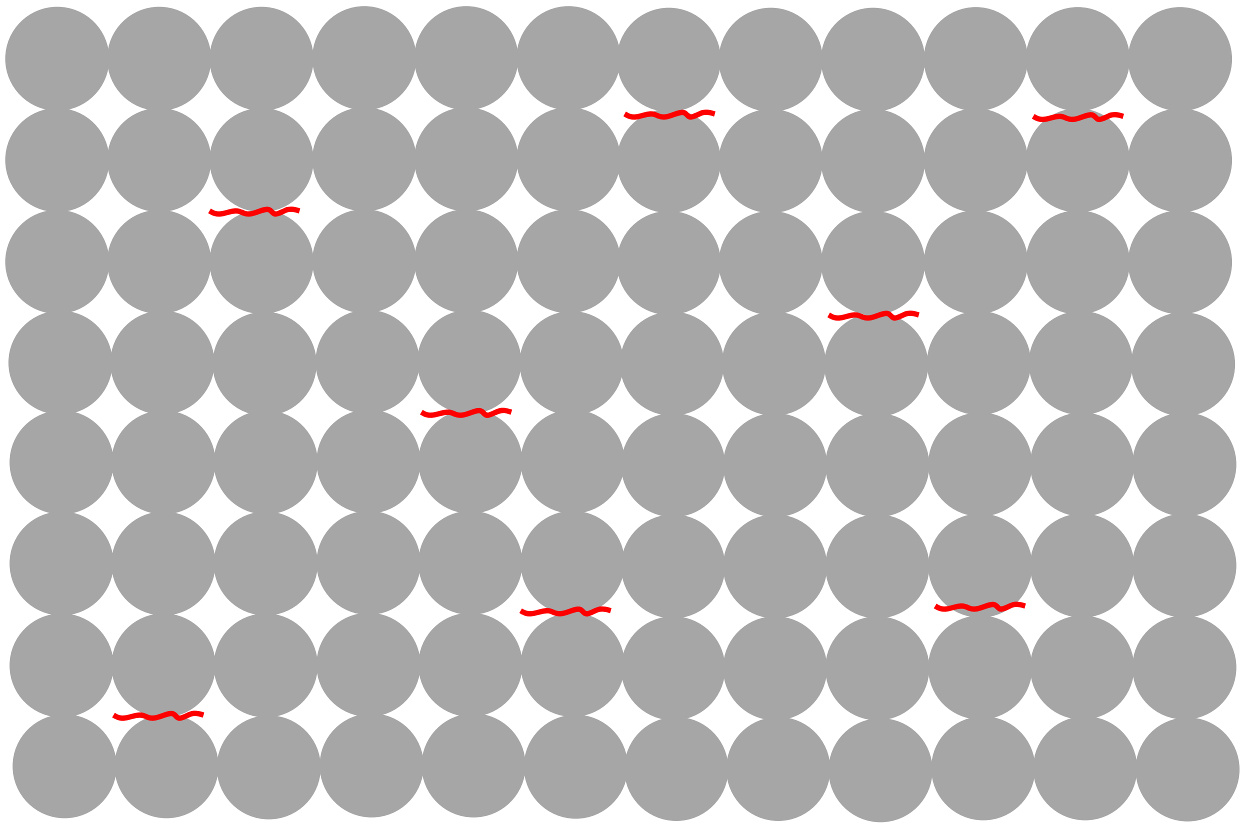
Crack – or failed grain contacts?



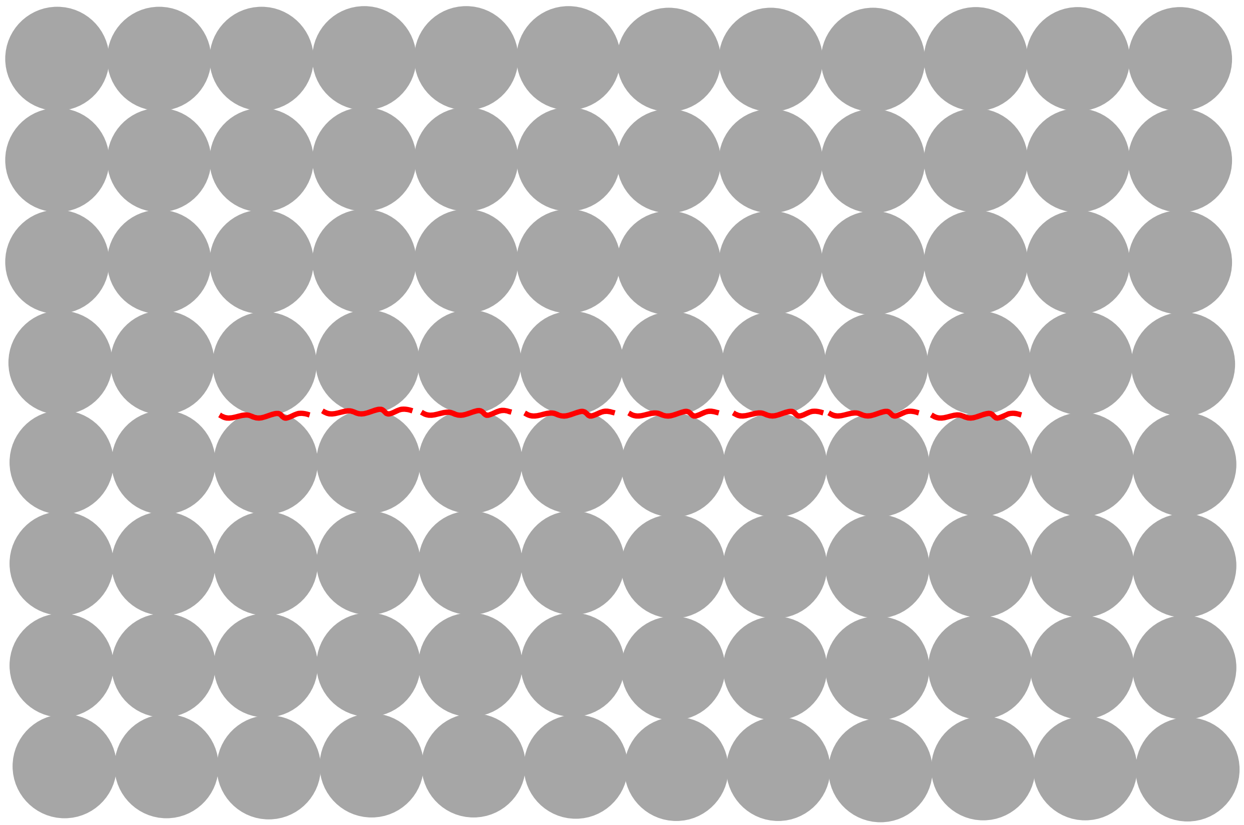
Crack – or failed grain contacts?



One large crack –

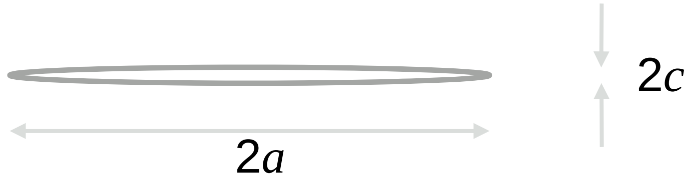


– or several failed grain contacts?



An assembly of failed grain contacts = a large crack has a much stronger impact on rock stiffness than the sum of the individual failed grain contacts

Open, flat cracks



Crack density: $\xi = n\langle a^3 \rangle$

n = number of cracks per unit volume

Isotropic distribution of cracks:

$$K^* = K_s\left(1 - \frac{16}{9}\frac{1-\nu_s^2}{1-2\nu_s}D\xi\right)$$

$$G^* = G_s\left(1 - \frac{32}{45}(1-\nu_s)\left[D + \frac{3}{2-\nu_s}\right]\xi\right)$$

Not consistent with Biot, but
– we may use the model to predict the properties of the dry material, which gives us the *frame moduli*.

Drainage parameter

$$\frac{1}{D} = 1 + \frac{4}{3\pi\gamma}\frac{1-\nu_s^2}{1-2\nu_s}\frac{K_f}{K_s}$$

Flat cracks: $\gamma = c/a \to 0$

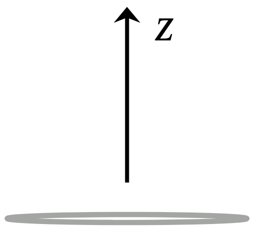
$\Rightarrow$

$D \to 0$ for saturated rock

$D = 1$ for dry rock

Non-isotropic distribution of cracks ⇒ **anisotropy**

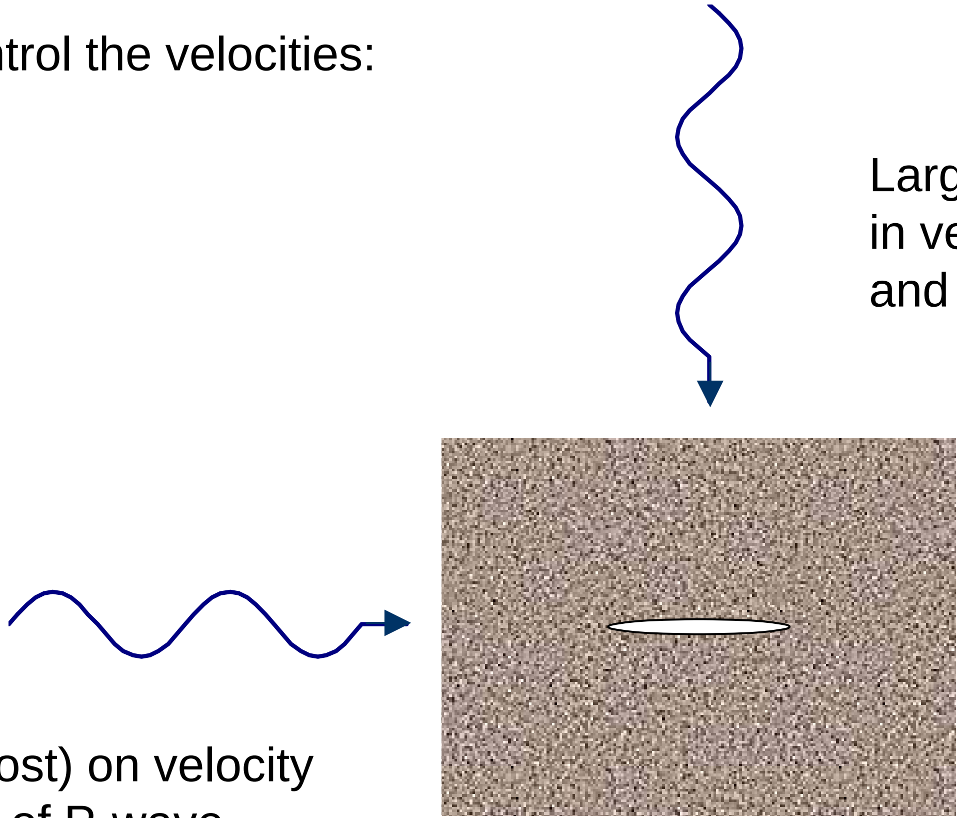
$$C_{ij}^{*} = C_{ij}^{o}\left(1 - \sum_{k} Q_{ij}^{k}\zeta_{k}\right)$$



$$Q^{k} = \frac{16}{3}\begin{bmatrix} \frac{\nu_s^2}{1-2\nu_s}D & \frac{\nu_s(1-\nu_s)}{1-2\nu_s}D & \frac{(1-\nu_s)^2}{1-2\nu_s}D & 0 & 0 & 0 \\ \frac{\nu_s(1-\nu_s)}{1-2\nu_s}D & \frac{\nu_s^2}{1-2\nu_s}D & \frac{(1-\nu_s)^2}{1-2\nu_s}D & 0 & 0 & 0 \\ \frac{(1-\nu_s)^2}{1-2\nu_s}D & \frac{(1-\nu_s)^2}{1-2\nu_s}D & \frac{(1-\nu_s)^2}{1-2\nu_s}D & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1-\nu_s}{2-\nu_s} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-\nu_s}{2-\nu_s} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Non-isotropic distribution of cracks ⇒ **anisotropy**

Cracks control the velocities:



Large reduction
in velocity
and amplitude

No effect (almost) on velocity
and amplitude of P-wave

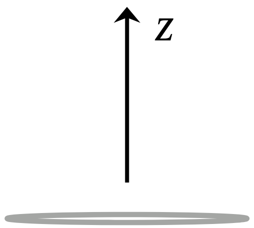
Some effect on vertically
polarized S-wave

Cracks can explain……….

	Propagation	*Polarization*	*Crack orientation*	*Velocity reduction*
P-wave	→	↔	⬯	Very strong
P-wave	→	↔	⬭	Weak
P-wave	→	↔	○	Weak
S-wave	→	↕	⬯	Strong
S-wave	→	↕	⬭	Strong
S-wave	→	↕	○	None

No drainage $\Rightarrow D = 0 \Rightarrow$ (nearly) no P-wave anisotropy

$$C_{ij}^{*} = C_{ij}^{o}\left(1 - \sum_{k} Q_{ij}^{k}\zeta_{k}\right)$$



$$Q^{k} = \frac{16}{3}\begin{bmatrix}
\frac{\nu_s^2}{1-2\nu_s}D & \frac{\nu_s(1-\nu_s)}{1-2\nu_s}D & \frac{(1-\nu_s)^2}{1-2\nu_s}D & 0 & 0 & 0 \\
\frac{\nu_s(1-\nu_s)}{1-2\nu_s}D & \frac{\nu_s^2}{1-2\nu_s}D & \frac{(1-\nu_s)^2}{1-2\nu_s}D & 0 & 0 & 0 \\
\frac{(1-\nu_s)^2}{1-2\nu_s}D & \frac{(1-\nu_s)^2}{1-2\nu_s}D & \frac{(1-\nu_s)^2}{1-2\nu_s}D & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1-\nu_s}{2-\nu_s} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1-\nu_s}{2-\nu_s} & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

"Saturation eliminates P-wave anisotropy"

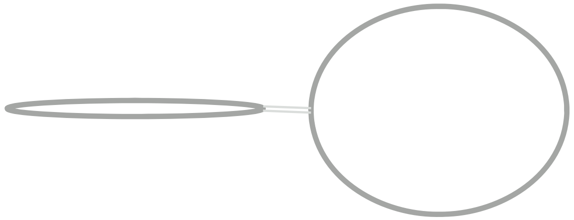
Leon Thomsen (1995):

Pore pressure equalization between cracks and pores

$$\frac{1}{D_{cp}} = 1 + \left[\frac{3}{2}\frac{1-\nu_s}{1-2\nu_s}\frac{\phi_p}{\phi} + \frac{4}{3\pi\gamma}\frac{1-\nu_s^2}{1-2\nu_s}\left(1-\frac{\phi_p}{\phi}\right)\right]\frac{K_f}{K_s}$$

$$\phi - \phi_p = \frac{4}{3}\pi\gamma\xi$$

Consequence: the drainage parameter will not vanish, regardless how thin the cracks are



"Saturation *does not* eliminate P-wave anisotropy in porous and permeable rocks"

General formalism for *displacement discontinuities* (*Sayers and Kachanov, 1995*):

$$S^*_{ijkl} = S^o_{ijkl} + \frac{1}{4}(\delta_{ik}\alpha_{jl} + \delta_{il}\alpha_{jk} + \delta_{jk}\alpha_{il} + \delta_{jl}\alpha_{ik}) + \beta_{ijkl}$$

$$\alpha_{ij} = \frac{1}{V}\sum_r B^r_T n^r_i n^r_j S_r$$

$$\beta_{ijkl} = \frac{1}{V}\sum_r (B^r_N - B^r_T) n^r_i n^r_j n^r_k n^r_l S_r$$

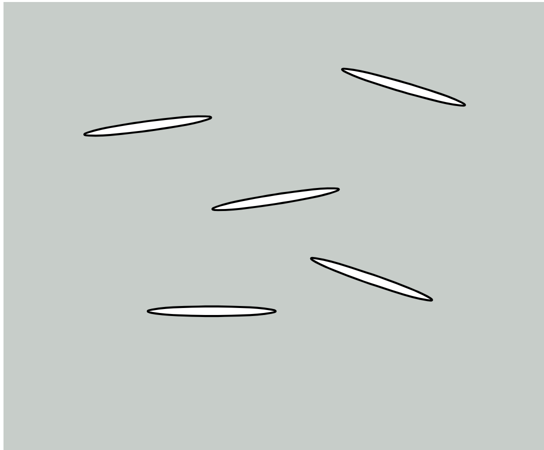
For open, penny-shaped cracks $B_N \simeq B_T \quad \Rightarrow \quad \beta_{ijkl} \simeq 0$

This approximation is not valid in general,
hence the open crack model is too simple.

However, we may compensate for this by allowing the drainage parameter D be an adjustable parameter.

Many cracks ⇒ crack interactions

- the presence of one crack may affect the influence of another



$$K^* = K_s\left(1 - Q\left(\nu_s\right)\zeta\right)$$

Self-consistent models:
Interactions are taken into account by giving the rock around a crack the properties of the effective medium

$$K^* = K_s - K^* Q\left(\nu_s^*\right)\zeta$$

⇒

$$K^* = \frac{K_s}{1 + Q\left(\nu_s^*\right)\zeta}$$

Alternative, equally valid procedures

$$K^* = K_s\left(1 - Q\left(\nu_s\right)\zeta\right)$$

$$\frac{1}{K^*} = \frac{1}{K_s} + \frac{Q\left(\nu_s\right)\zeta}{K_s}$$

Self-consistency:

$$K^* = K_s - K^* Q\left(\nu^*\right)\zeta$$

$$\frac{1}{K^*} = \frac{1}{K_s} + \frac{Q\left(\nu^*\right)\zeta}{K^*}$$

$$\Downarrow$$

$$K^* = \frac{K_s}{1 + Q\left(\nu^*\right)\zeta}$$

$$K^* = K_s\left(1 - Q\left(\nu^*\right)\zeta\right)$$

$$K^* > 0 \quad \text{always}$$

$$K^* = 0 \;\; \text{for} \quad \zeta = 1/Q\left(\nu^*\right)$$

There are many different self-consistent models,
giving different predictions – depending on the initial model.

The Differential Effective Medium (DEM) model resolves this discrepancy by adding small numbers of cracks in many steps, and recalculating the effective stiffness for each step (always working in the non-interacting limit)

$$dK^{*} = -K^{*} Q\left(\nu^{*}\right) d\zeta$$

This gives the same (*unique*!) solution for all initial models.

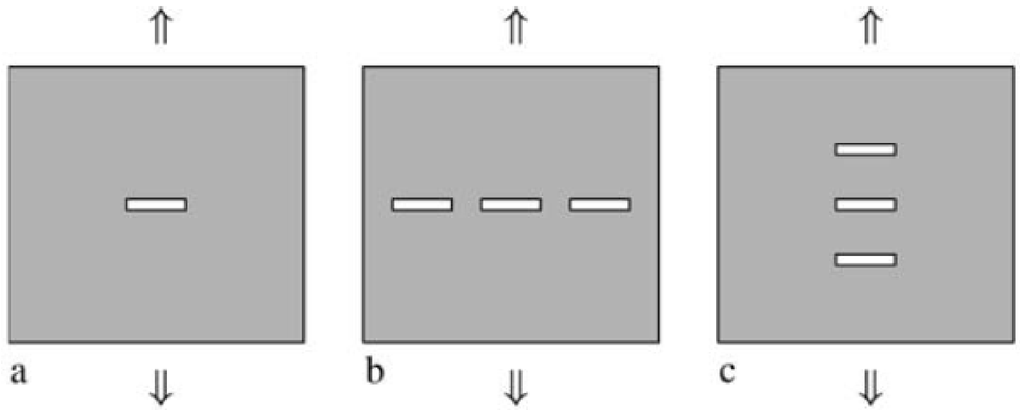
But – is it more correct because of that?

Models accounting for interactions

Many alternatives…..
- all are mathematically correct, but they give different predictions.

Which one is correct?

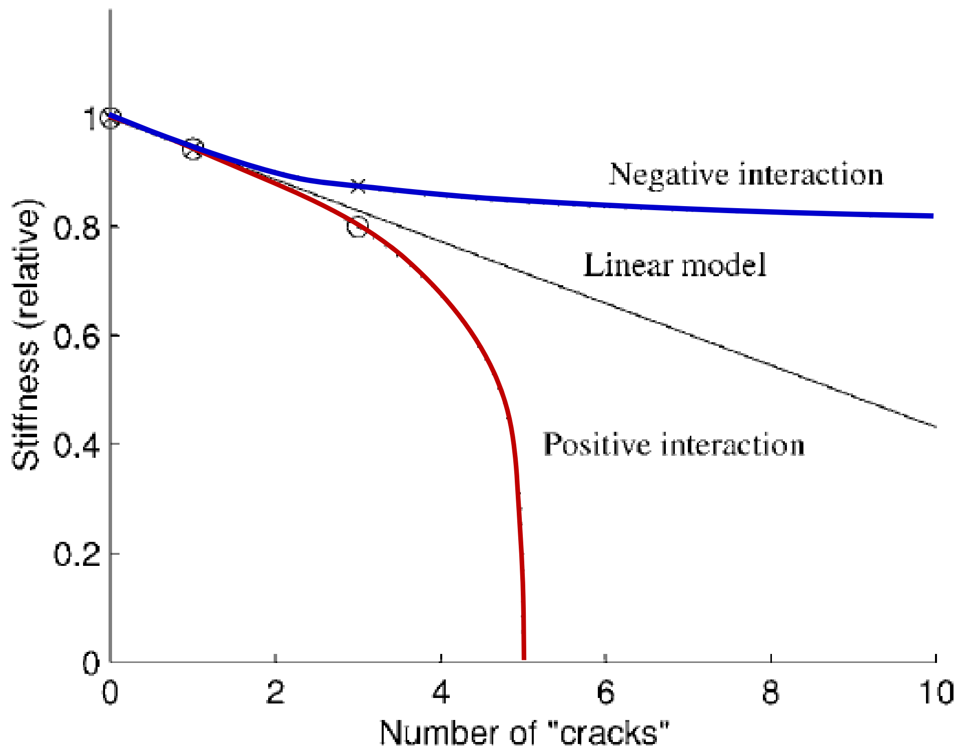
It depends….



The actual position of the cracks relative to each other determines which model is the correct one to use.

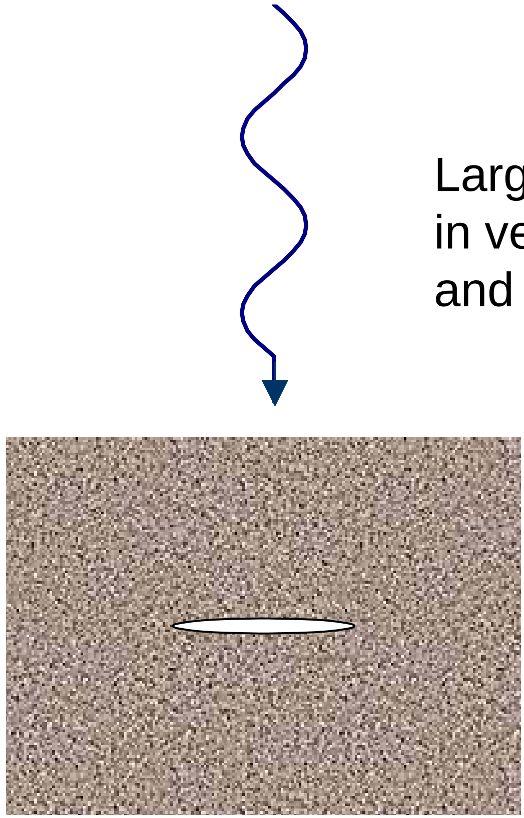
Unfortunately, no model comes with a description of how the cracks are positioned.

If we do not know
how the cracks are positioned
(and we usually don't'),
the linear, non-interacting model
may be a good starting point.

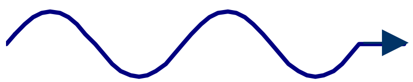


Stress effects on the rock framework

Cracks control the velocities:



Large reduction
in velocity
and amplitude

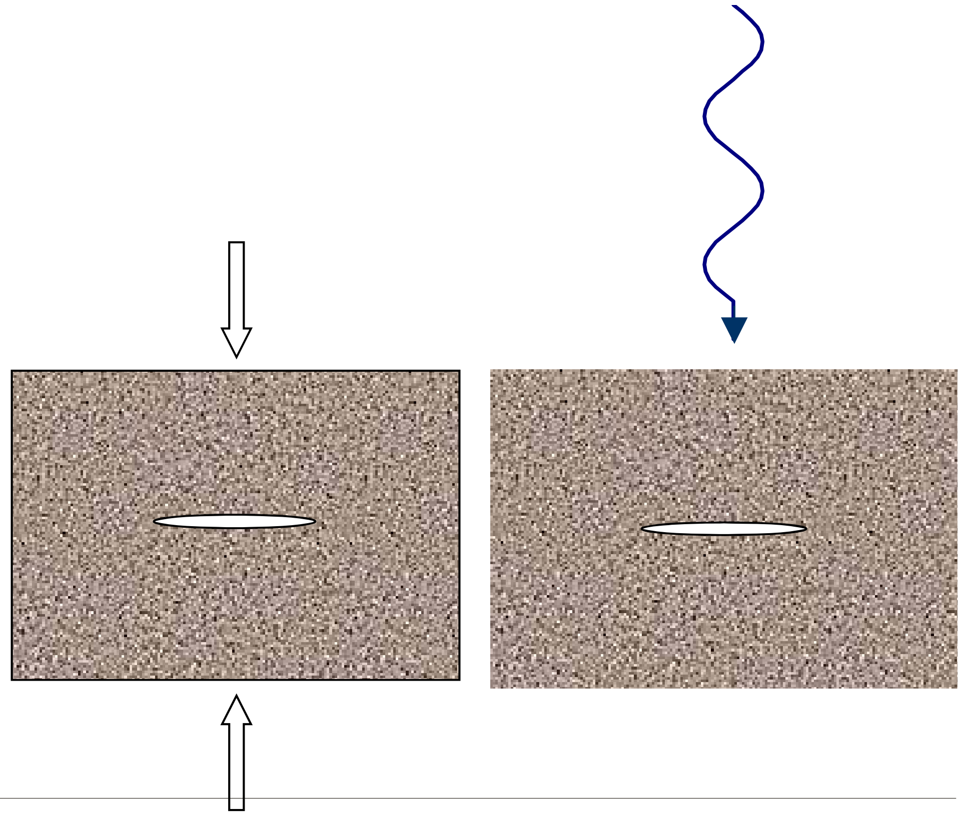


No effect (almost) on velocity
and amplitude of P-wave

Some effect on vertically
polarized S-wave

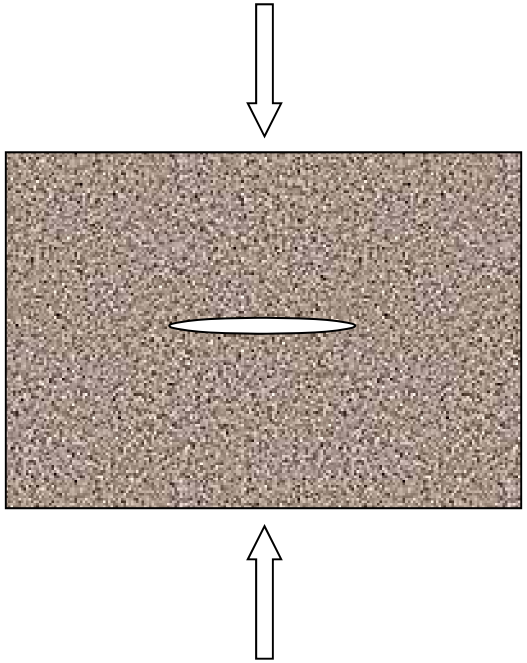
Rule of thumb:

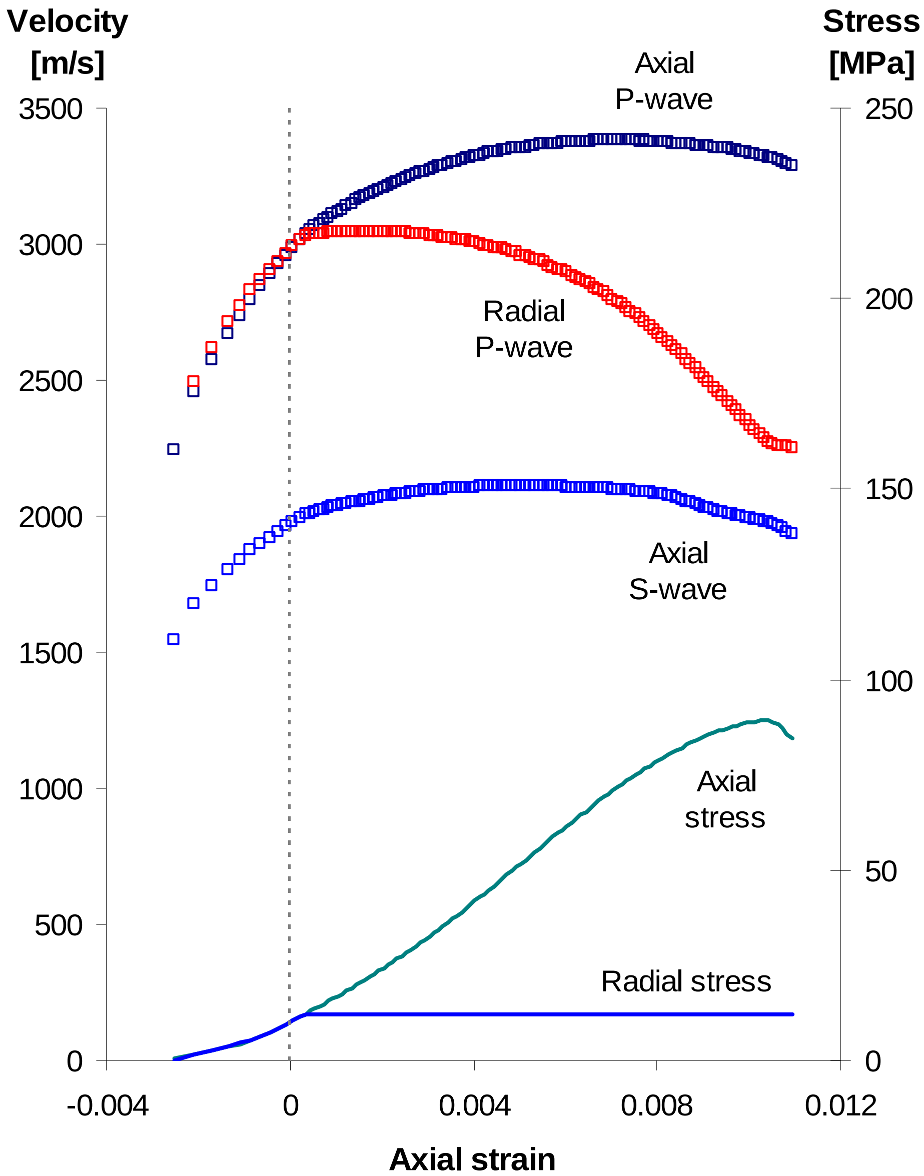
Velocities are mostly affected by changes in the normal stress in the direction of propagation (and polarization)



Cracks are sensitive to changes in stress

A compressive principal stress tends to close a crack that is oriented normal to the stress





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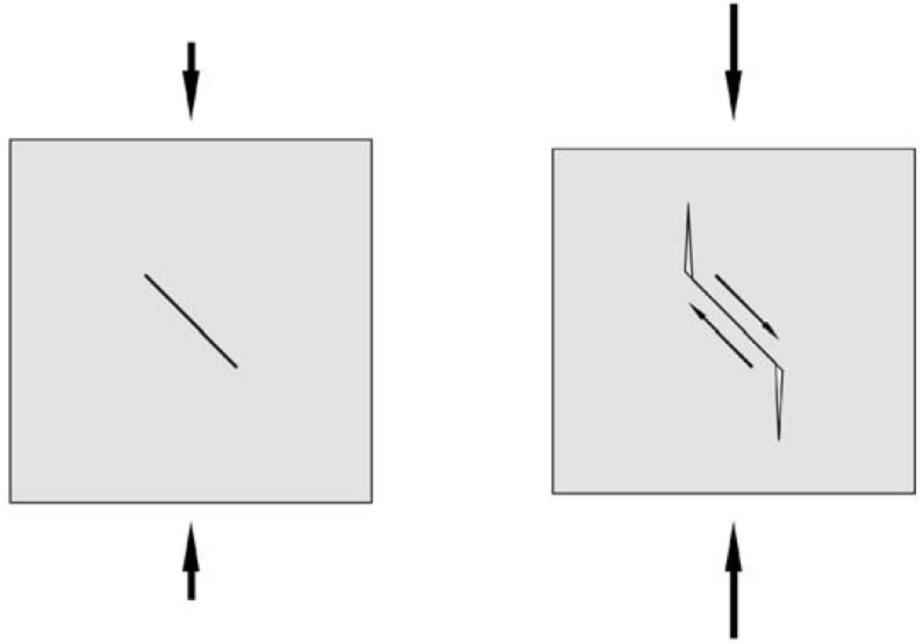
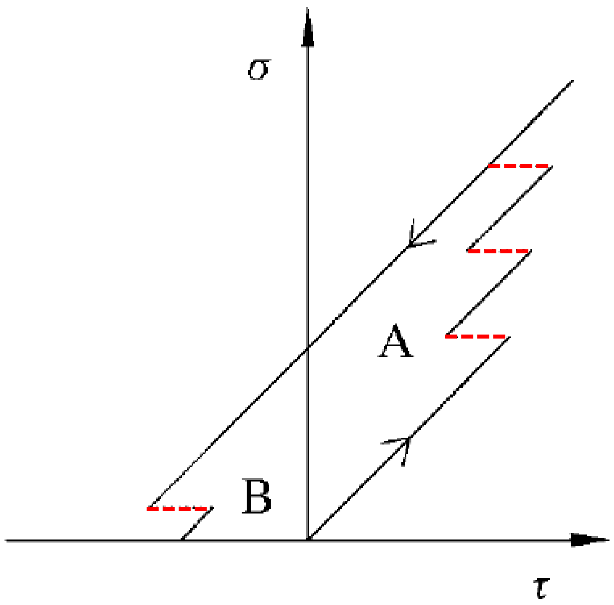
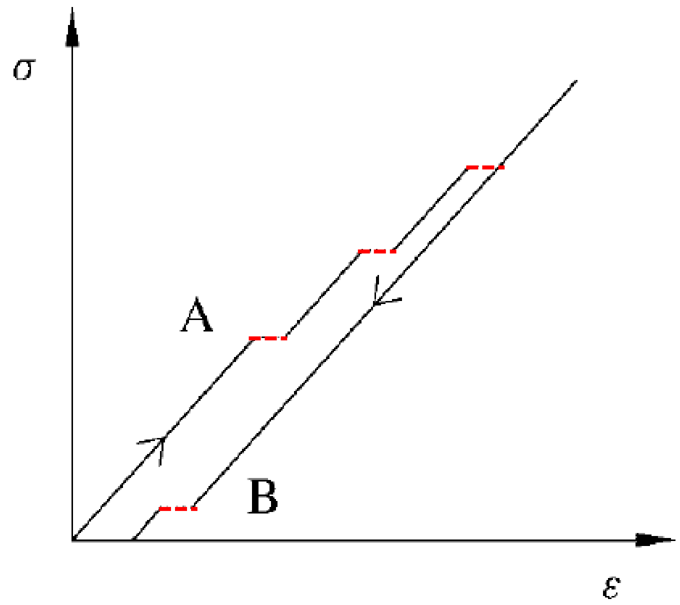
Sliding cracks



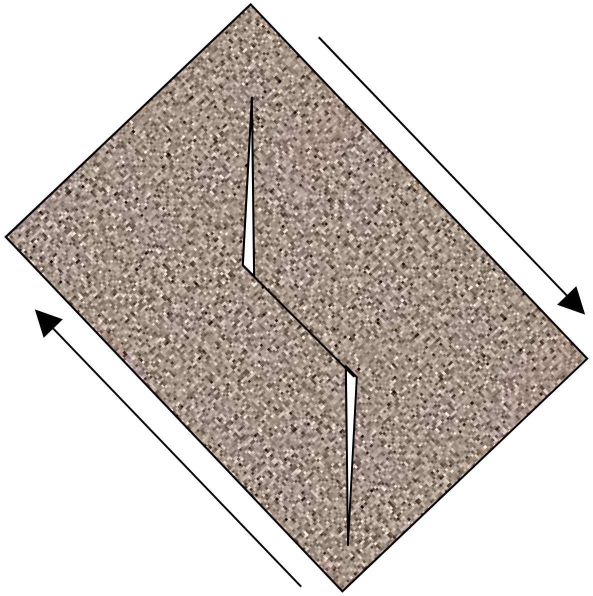
- induce hysteresis, permanent deformation, and difference between loading and unloading modulus

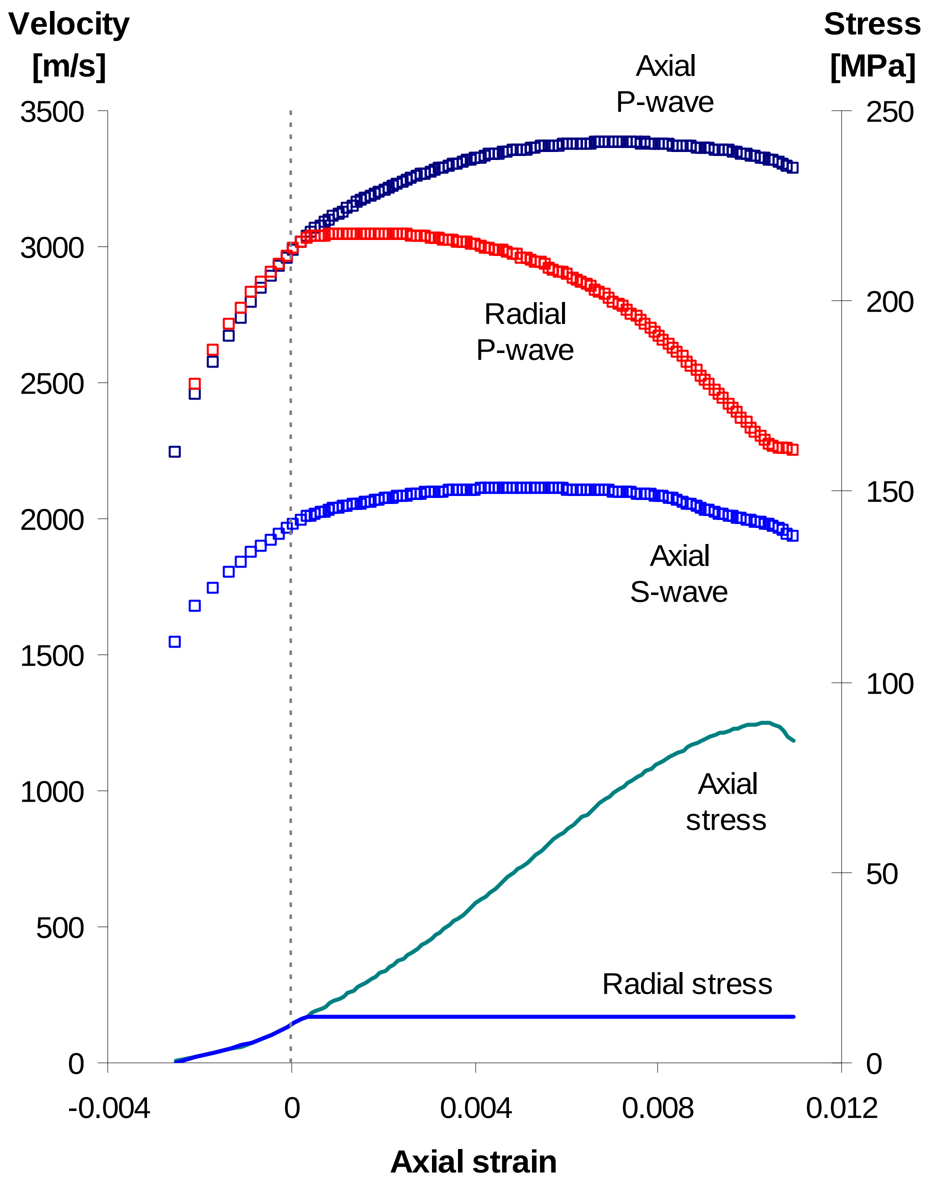




Cracks are sensitive to changes in stress

Shear deformations tend to open up cracks





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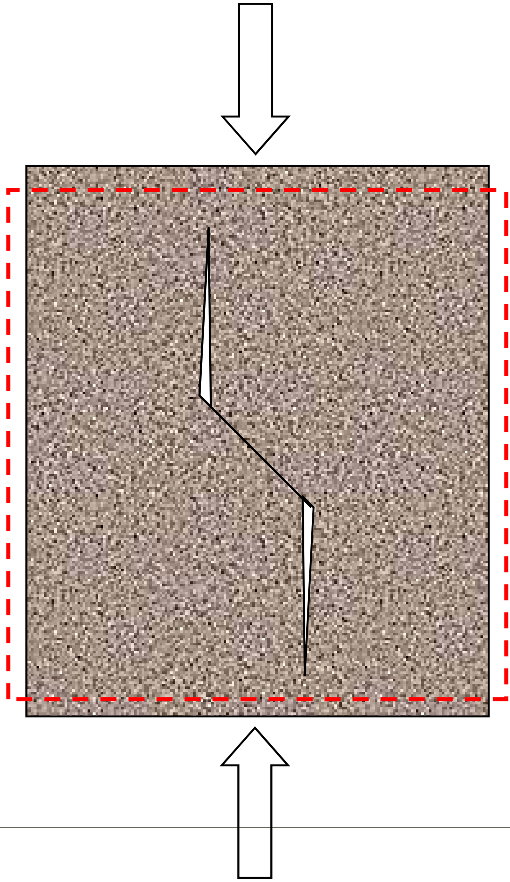

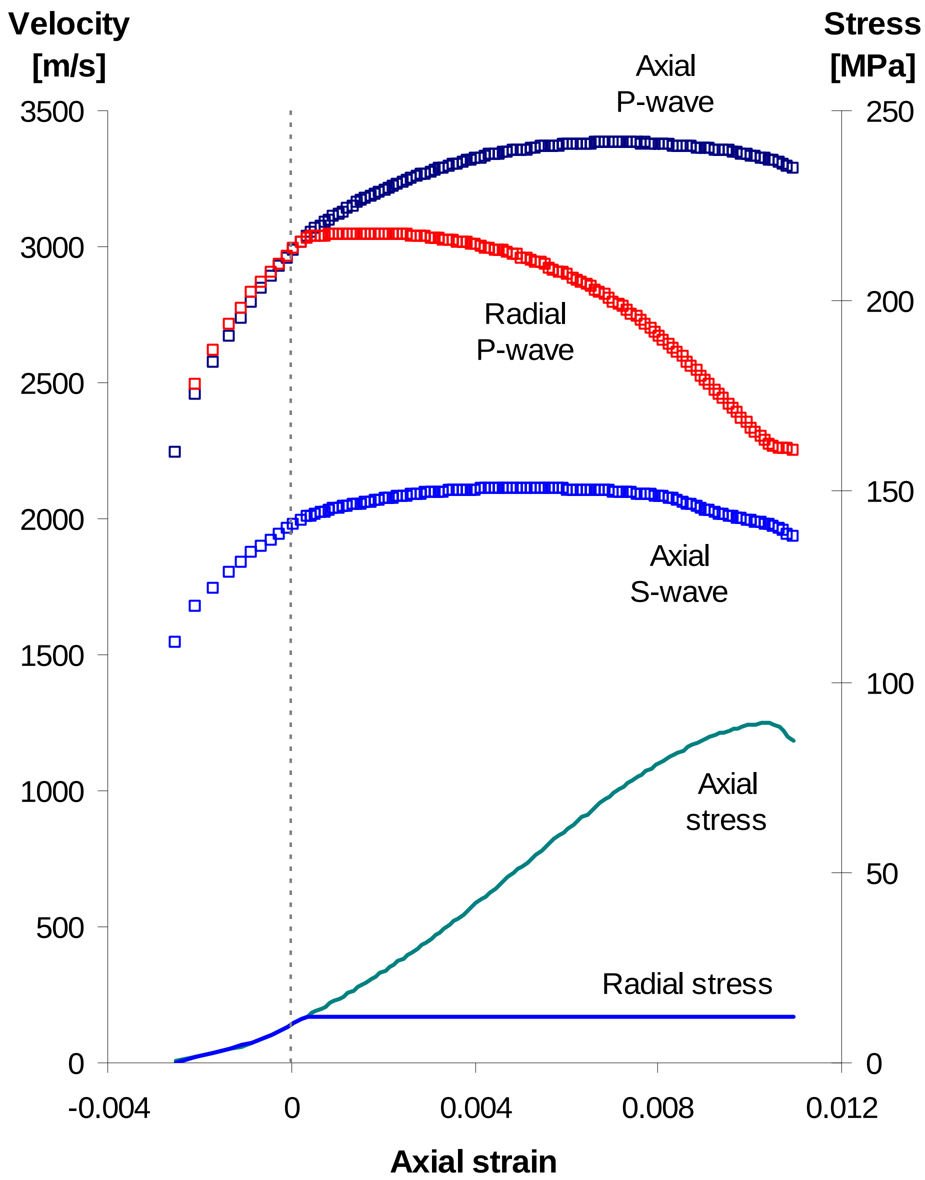

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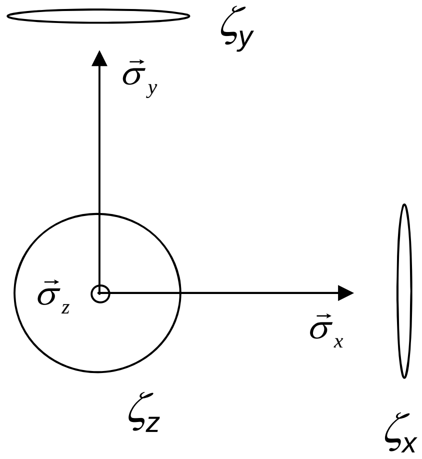
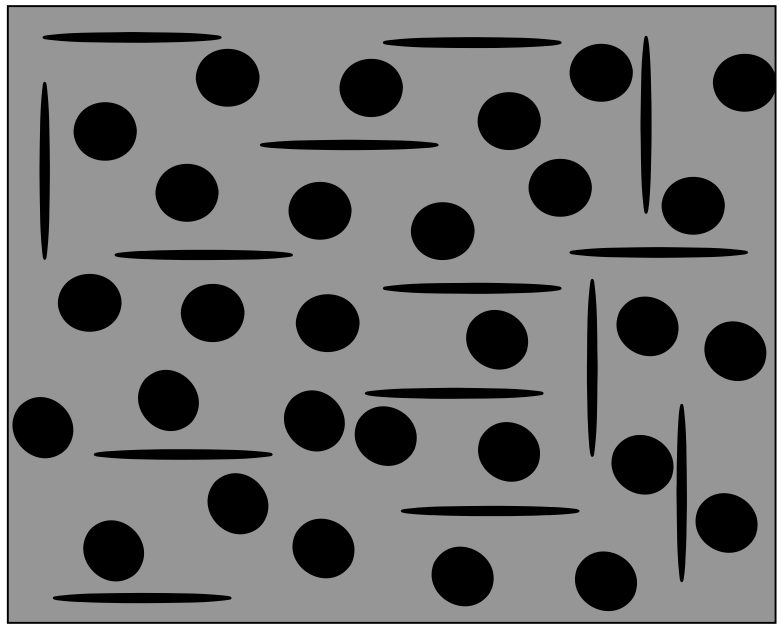
Fjær (2006):

A model

Three sets of flat cracks
oriented normal to
the principal stresses

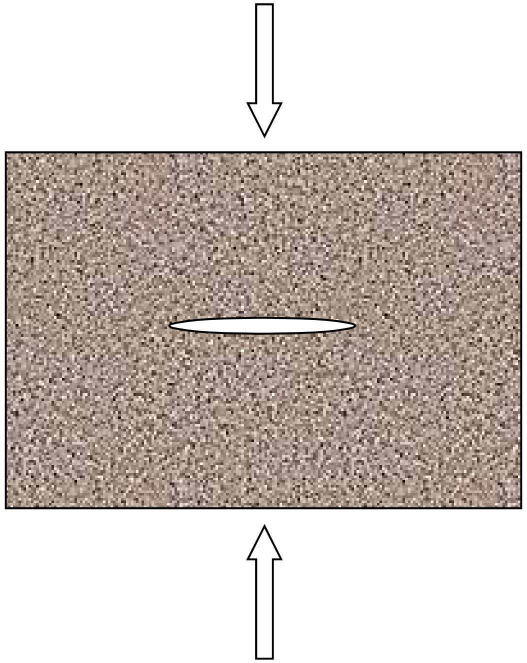




Solid, pores & cracks

Cracks are sensitive to changes in stress

A compressive principal stress tends to close a crack that is oriented normal to the stress



We can not close a crack that is already closed

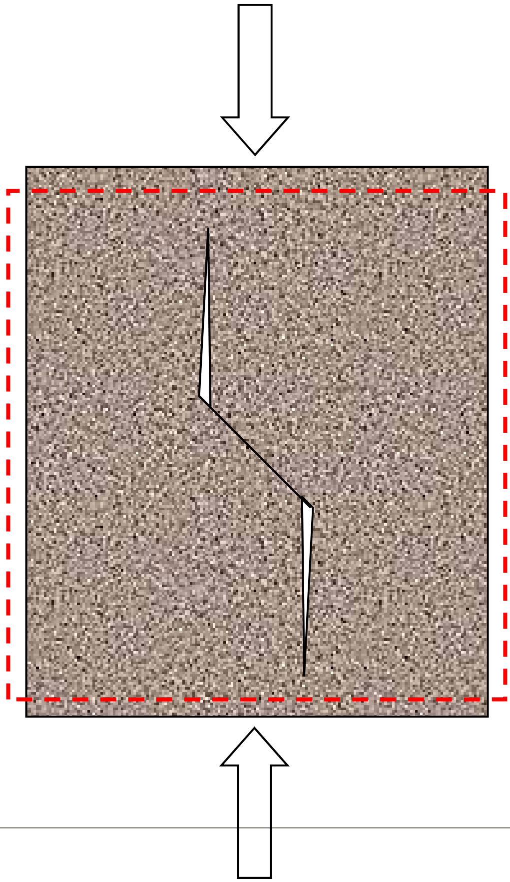
$$d\zeta_i \propto -\zeta_i d\sigma_i$$

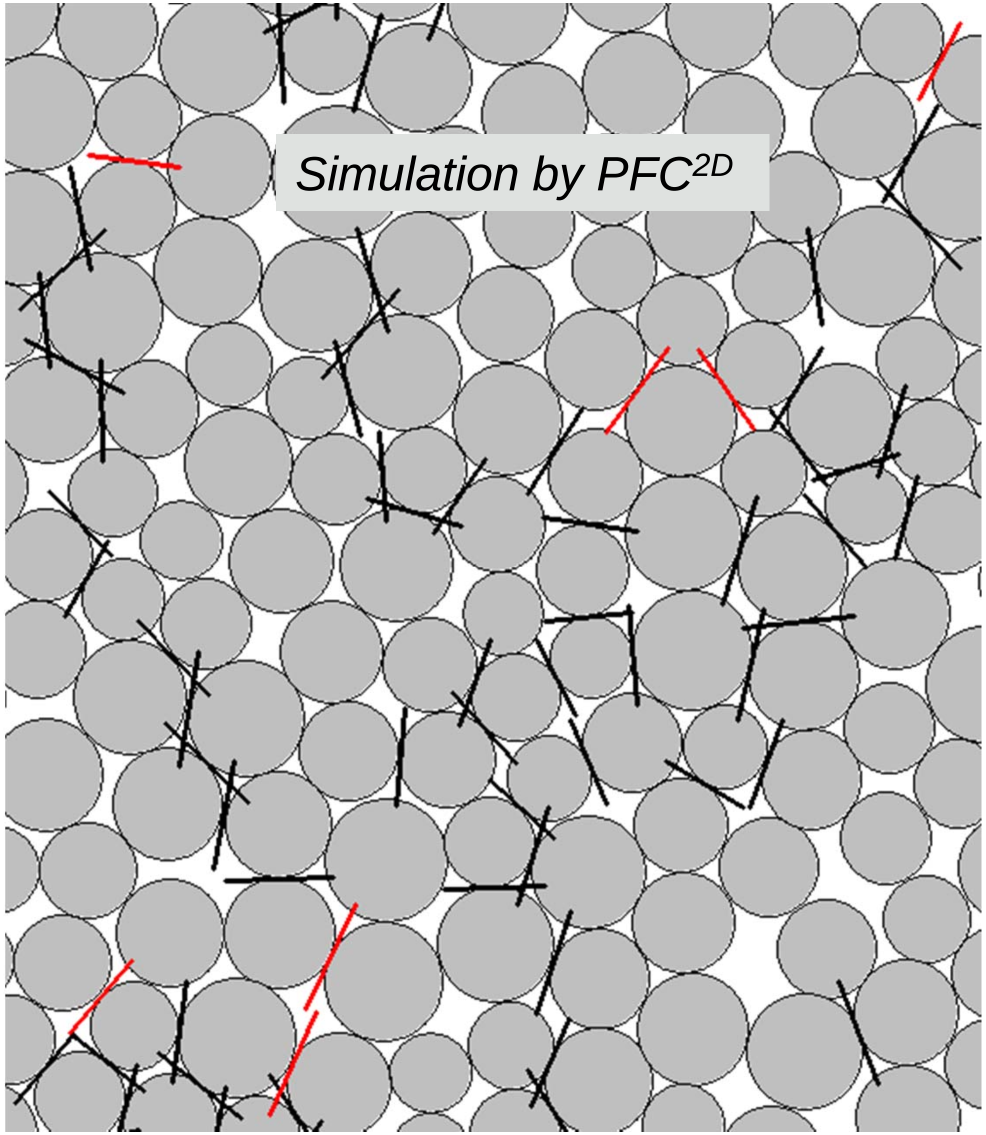
Assumption:

$$\zeta_i \propto \left(\sigma_i + T_o\right)^{-n}$$

Cracks are sensitive to changes in stress

Shear deformations
tend to open up cracks





Local failures induced by compressive stress:

shear ———
tensile ———

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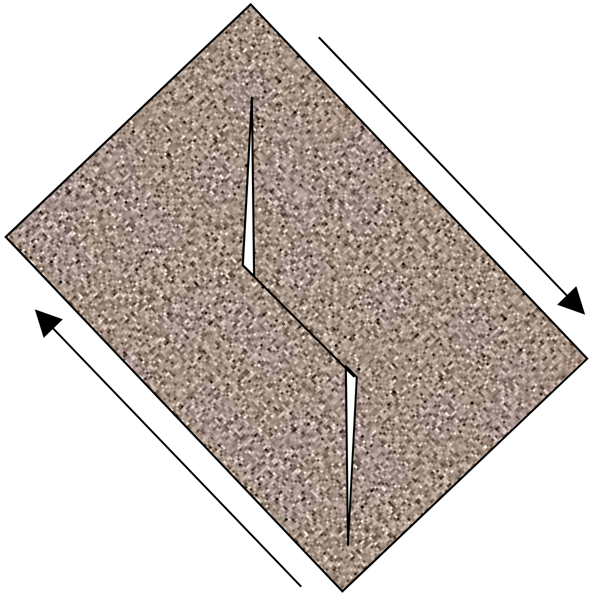
Cracks are sensitive to changes in stress

Shear deformations
tend to open up cracks



Sensitivity to shear strain:

$$d\zeta_i \propto -\left(d\varepsilon_i - d\varepsilon_j\right) - \left(d\varepsilon_i - d\varepsilon_k\right)$$

Assumption:

$$\zeta_i \propto e^{-\beta\left(2\varepsilon_i - \varepsilon_j - \varepsilon_k\right)}$$

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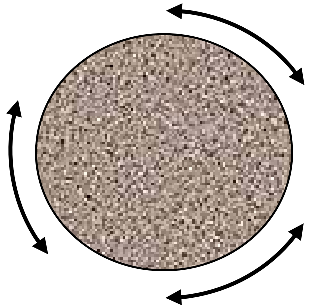
Very large shear strains

$\Rightarrow$ more turbulent crack development

Changes in crack density more sensitive to magnitude than to orientation of shear strain



Assumption:

$$\zeta_i \propto e^{\eta \Gamma^2}$$

Γ = maximum shear strain

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Mathematics of the model:

$$C_{11} = C_{11}^{o}\left[1 - Q_{11}^{p}\phi - Q_{33}\zeta_{x} - Q_{11}\left(\zeta_{y} + \zeta_{z}\right)\right]$$ etc.

$$\zeta_{i} = \zeta_{i}^{o}\left(\frac{\sigma_{i}^{o} + T_{o}}{\sigma_{i} + T_{o}}\right)^{n} e^{-\beta\left(2\varepsilon_{i} - \varepsilon_{j} - \varepsilon_{k}\right) + \eta\Gamma^{2}}$$

$$\phi = \frac{\phi_{o} - \varepsilon_{v}}{1 - \varepsilon_{v}}$$

Velocities:

$$V_{p1} \quad \left(= \sqrt{\frac{C_{11}}{\rho}}\right)$$ etc.

Assumptions:

$$C_{11}^{o} = C_{22}^{o} = C_{33}^{o} = H_{o}$$

$$C_{44}^{o} = C_{55}^{o} = C_{66}^{o} = G_{o}$$

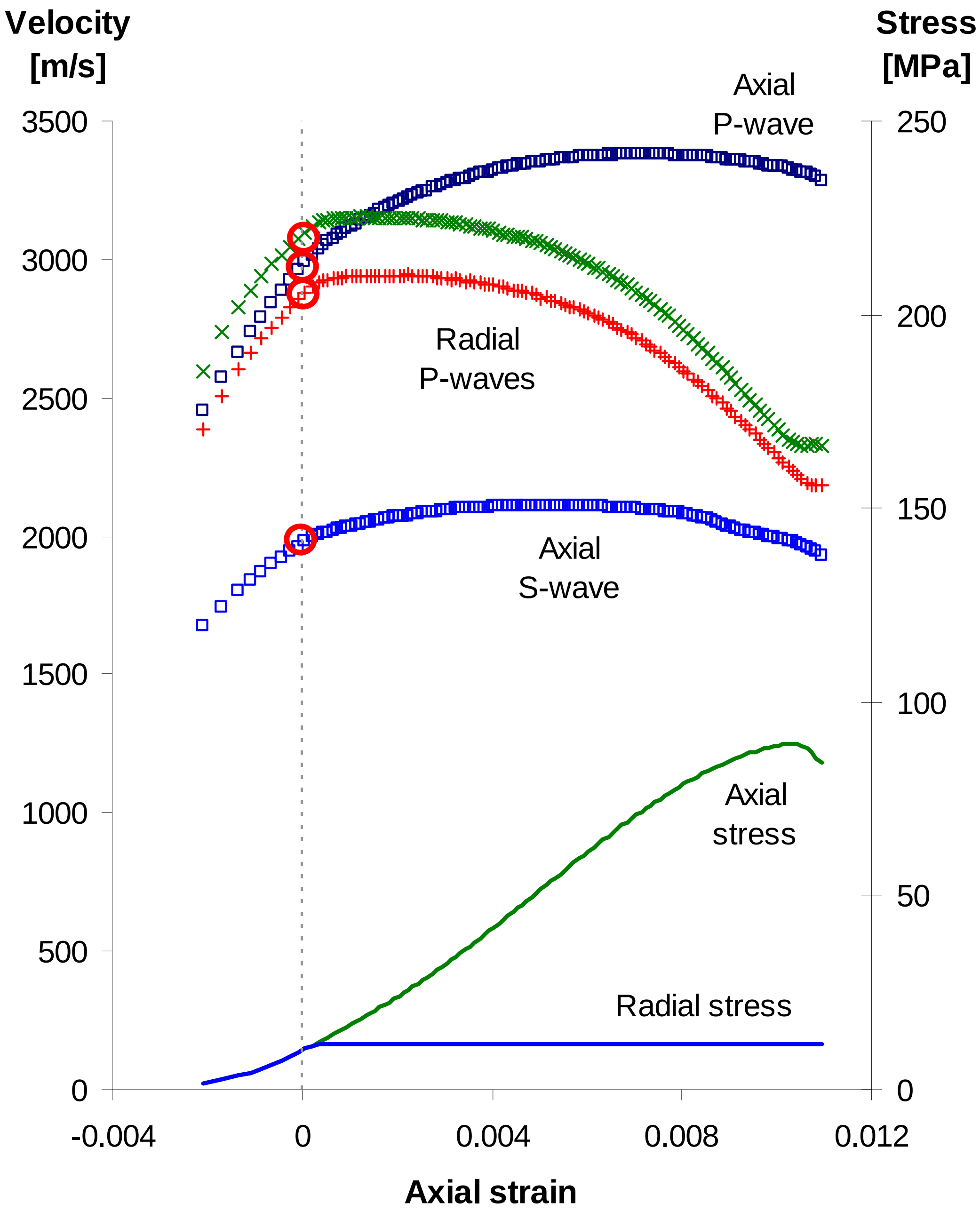
$$C_{12}^{o} = C_{13}^{o} = C_{23}^{o} = H_{o} - 2G_{o}$$

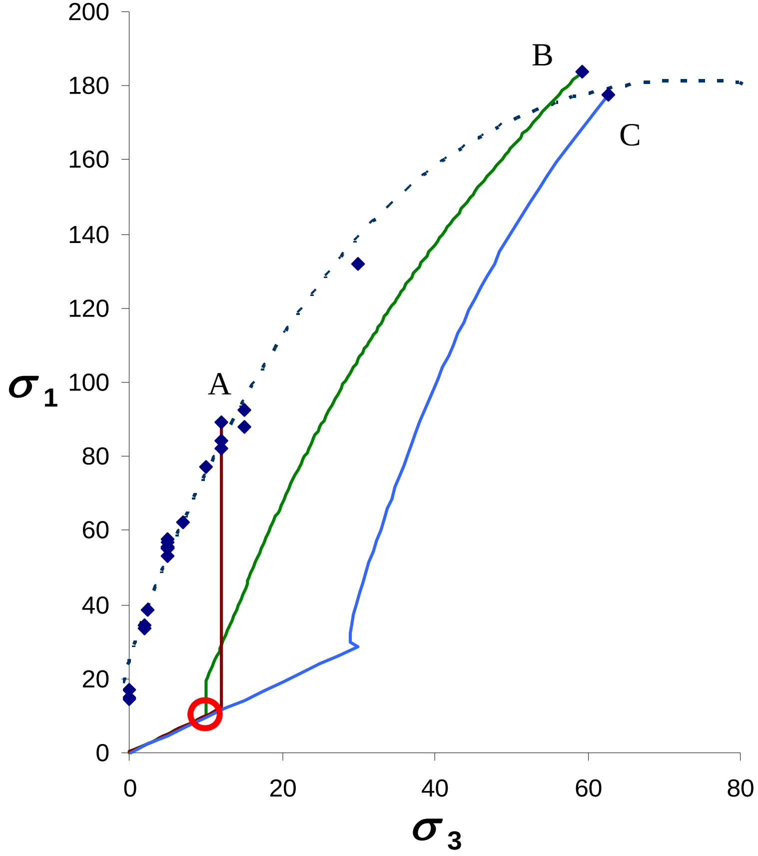
$H_{o} = 80$ GPa (fixed value)

$\nu = 0.2$ (fixed value)

Test A

Scaling: $\zeta_x^o, \zeta_y^o, \zeta_z^o, G_o$





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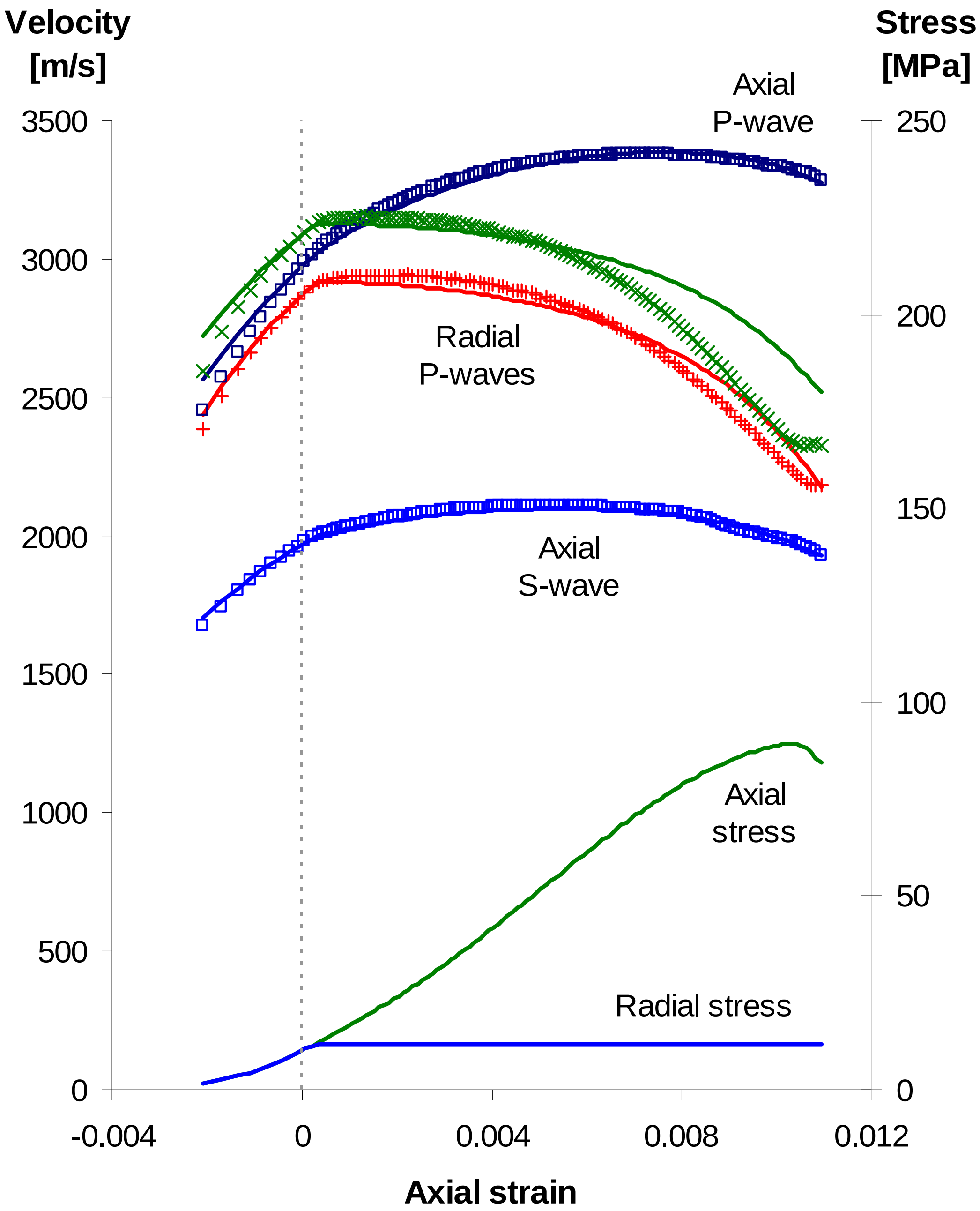
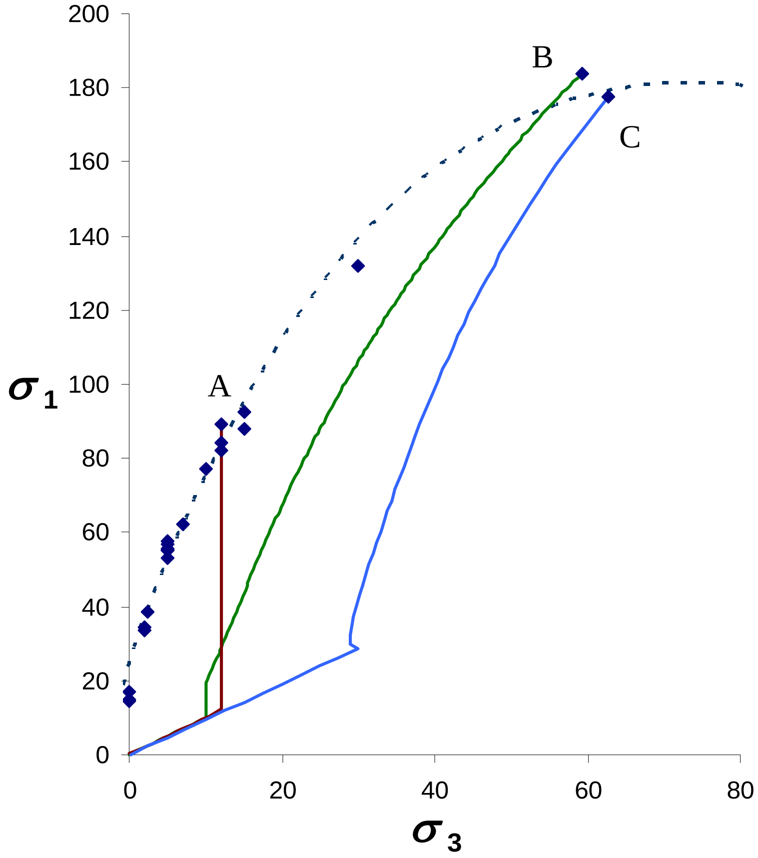


Test A

$n = 0.1$

$\beta = 5$

$\eta = 800$



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$\Rightarrow$

The model – based on flat cracks and sperical pores - matches observations quite well

The match supports the claim that the stress dependency of wave velocities may largely be explained in terms of opening and closure of cracks

References:

Fjær, E., Holt, R.M., Horsrud, P., Raaen, A.M. and Risnes, R. (2008) "Petroleum Related Rock Mechanics. 2nd Edition". Elsevier, Amsterdam

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