Contrast-source inversion of CSEM and MT data

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Introduction

Contrast-source inversion

- Whenever you have a Lippmann-Schwinger equation
- Developed by P. van den Berg and co-workers (Abubakar and van den Berg, Inverse Problems, 2002)

Some applications:

- Medical acoustic imaging: van den Berg et al.
- Ground-penetrating radar: Feng et al (2003)
- Diffusive EM fields: Wiik et al. (2011)

Publications

- Wiik, T. and Hokstad, K., 2008, **Contrast-Source inversion of mCSEM data for hydrocarbon prospecting**: EAGE, Rome 2008.

- Wiik,T., Løseth,L., Ursin,B. and Hokstad,K, 2011, **TIV contrast source inversion of mCSEM data**: Geophysics 76, F65-F76

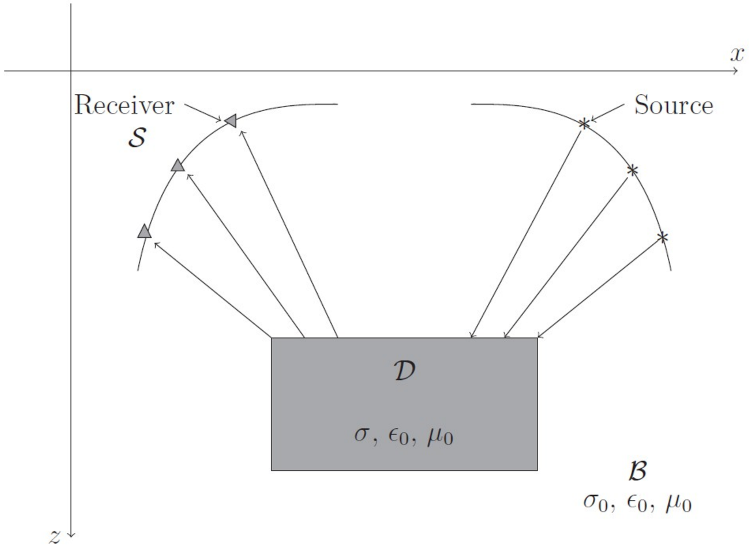
- Wiik,T., Hokstad,K. and Ursin,B., Hokstad,K: **Joint inversion of mCSEM and MT data**: Submitted to Geophysical Prosepecting

Background and anomaly

Anomalous conductivity

$$\chi_h = \frac{\sigma_h}{\sigma_{0,v}} - \Upsilon \qquad \chi_v = \frac{\sigma_v}{\sigma_{0,v}} - 1$$

$$\boldsymbol{\chi} = \begin{pmatrix} \chi_h & 0 & 0 \\ 0 & \chi_h & 0 \\ 0 & 0 & \chi_v \end{pmatrix}$$



Lippmann-Schwinger

$$e_i(\mathbf{x}) = e_i^{\mathrm{inc}}(\mathbf{x}) + \int_{\mathcal{D}} G_{ij}^{E}(\mathbf{x}, \mathbf{x}')\, \sigma_{0,v}(\mathbf{x}')\, \chi_{jj}(\mathbf{x}')\, e_j(\mathbf{x}')\, \mathrm{d}\mathbf{x}', \quad \mathbf{x} \in \mathcal{D}$$

Scattered E and H fields

Data equation

$$
\begin{aligned}
f_i^E(\mathbf{x}) &= \int_{\mathcal{D}} G_{ij}^E(\mathbf{x},\mathbf{x}')\,\sigma_{0,v}(\mathbf{x}')\,\chi_{jj}(\mathbf{x}')\,e_j(\mathbf{x}')\,\mathrm{d}\mathbf{x}', \quad \mathbf{x}\in\mathcal{S} \\
f_i^H(\mathbf{x}) &= \int_{\mathcal{D}} G_{ij}^H(\mathbf{x},\mathbf{x}')\,\sigma_{0,v}(\mathbf{x}')\,\chi_{jj}(\mathbf{x}')\,e_j(\mathbf{x}')\,\mathrm{d}\mathbf{x}', \quad \mathbf{x}\in\mathcal{S}
\end{aligned}
$$

(Lippmann-Schwinger again)

Operator notation

$$\mathbf{e} = \mathbf{e}^{\mathrm{inc}} + G^{E,\mathcal{D}} \boldsymbol{\chi} \mathbf{e},$$

Lippmann-Schwinger on anomaly

$$\mathbf{f}^{E} = G^{E,\mathcal{S}} \boldsymbol{\chi} \mathbf{e},$$

Scattered E-field on receiver

$$\mathbf{f}^{H} = G^{H,\mathcal{S}} \boldsymbol{\chi} \mathbf{e},$$

Scattered H-field on receiver

Contrast sources

$$
\begin{aligned}
\mathcal{W}_{\text{CSEM}} &= \left\{\mathbf{w}_{\text{CSEM}}^{j,k}\right\}_{j=1\ldots N_s}^{k=1\ldots N_{f_{\text{CSEM}}}} = \left\{\chi\mathbf{e}_{\text{CSEM}}^{j,k}\right\}_{j=1\ldots N_s}^{k=1\ldots N_{f_{\text{CSEM}}}}\,, \\
\mathcal{W}_{\text{MT}} &= \left\{\mathbf{w}_{\text{MT}}^{k}\right\}^{k=1\ldots N_{f_{\text{MT}}}} = \left\{\chi\mathbf{e}_{\text{MT}}^{k}\right\}^{k=1\ldots N_{f_{\text{MT}}}}\,,
\end{aligned}
$$

Operators

$$
\begin{aligned}
\mathbf{e} &= \mathbf{e}^{\text{inc}} + G^{E,\mathcal{D}}\mathbf{w}, \\
\mathbf{f}^{E} &= G^{E,\mathcal{S}}\mathbf{w}, \\
\mathbf{f}^{H} &= G^{H,\mathcal{S}}\mathbf{w}.
\end{aligned}
$$

Objective function

$$
\begin{aligned}
F_1\left(\mathcal{W}_{\mathrm{CSEM}}, \mathcal{W}_{\mathrm{MT}}, \boldsymbol{\chi}\right) \;=\;& \alpha^{E}_{1,\mathrm{CSEM}} \sum_{k=1}^{N_{\mathrm{f}_{\mathrm{CSEM}}}} \sum_{j=1}^{N_{\mathrm{s}}} \left\| \Xi^{E,j,k}_{\mathrm{CSEM}} \left(\mathbf{f}^{E,j,k}_{\mathrm{CSEM}} - G^{E,\mathcal{S},k} \mathbf{w}^{j,k}_{\mathrm{CSEM}} \right) \right\|^2_{\mathcal{S}} \\
+\;& \alpha^{H}_{1,\mathrm{CSEM}} \sum_{k=1}^{N_{\mathrm{f}_{\mathrm{CSEM}}}} \sum_{j=1}^{N_{\mathrm{s}}} \left\| \Xi^{H,j,k}_{\mathrm{CSEM}} \left(\mathbf{f}^{H,j,k}_{\mathrm{CSEM}} - G^{H,\mathcal{S},k} \mathbf{w}^{j,k}_{\mathrm{CSEM}} \right) \right\|^2_{\mathcal{S}} \\
+\;& \alpha_{2,\mathrm{CSEM}} \sum_{k=1}^{N_{\mathrm{f}_{\mathrm{CSEM}}}} \sum_{j=1}^{N_{\mathrm{s}}} \left\| \boldsymbol{\chi} \mathbf{e}^{\mathrm{inc},j,k}_{\mathrm{CSEM}} - \mathbf{w}^{j,k}_{\mathrm{CSEM}} + \boldsymbol{\chi} G^{E,\mathcal{D},k} \mathbf{w}^{j,k}_{\mathrm{CSEM}} \right\|^2_{\mathcal{D}} \\
+\;& \alpha^{E}_{1,\mathrm{MT}} \sum_{k=1}^{N_{\mathrm{f}_{\mathrm{MT}}}} \left\| \Xi^{E,k}_{\mathrm{MT}} \left(\mathbf{f}^{E,k}_{\mathrm{MT}} - G^{E,\mathcal{S},k} \mathbf{w}^{k}_{\mathrm{MT}} \right) \right\|^2_{\mathcal{S}} \\
+\;& \alpha^{H}_{1,\mathrm{MT}} \sum_{k=1}^{N_{\mathrm{f}_{\mathrm{MT}}}} \left\| \Xi^{H,k}_{\mathrm{MT}} \left(\mathbf{f}^{H,k}_{\mathrm{MT}} - G^{H,\mathcal{S},k} \mathbf{w}^{k}_{\mathrm{MT}} \right) \right\|^2_{\mathcal{S}} \\
+\;& \alpha_{2,\mathrm{MT}} \sum_{k=1}^{N_{\mathrm{f}_{\mathrm{MT}}}} \left\| \boldsymbol{\chi} \mathbf{e}^{\mathrm{inc},k}_{\mathrm{MT}} - \mathbf{w}^{k}_{\mathrm{MT}} + \boldsymbol{\chi} G^{E,\mathcal{D},k} \mathbf{w}^{k}_{\mathrm{MT}} \right\|^2_{\mathcal{D}}
\end{aligned}
$$

Objective function

$$
\begin{aligned}
F_1\left(\mathcal{W}_{\mathrm{CSEM}}, \mathcal{W}_{\mathrm{MT}}, \chi\right) = \; & \alpha^{E}_{1,\mathrm{CSEM}} \sum_{k=1}^{N_{\mathrm{f_{CSEM}}}} \sum_{j=1}^{N_{\mathrm{s}}} \left\| \Xi^{E,j,k}_{\mathrm{CSEM}} \left(\mathbf{f}^{E,j,k}_{\mathrm{CSEM}} - G^{E,\mathcal{S},k} \mathbf{w}^{j,k}_{\mathrm{CSEM}} \right) \right\|^2_{\mathcal{S}} \\
+ \; & \alpha^{H}_{1,\mathrm{CSEM}} \sum_{k=1}^{N_{\mathrm{f_{CSEM}}}} \sum_{j=1}^{N_{\mathrm{s}}} \left\| \Xi^{H,j,k}_{\mathrm{CSEM}} \left(\mathbf{f}^{H,j,k}_{\mathrm{CSEM}} - G^{H,\mathcal{S},k} \mathbf{w}^{j,k}_{\mathrm{CSEM}} \right) \right\|^2_{\mathcal{S}} \\
+ \; & \alpha_{2,\mathrm{CSEM}} \sum_{k=1}^{N_{\mathrm{f_{CSEM}}}} \sum_{j=1}^{N_{\mathrm{s}}} \left\| \chi \mathbf{e}^{\mathrm{inc},j,k}_{\mathrm{CSEM}} - \mathbf{w}^{j,k}_{\mathrm{CSEM}} + \chi G^{E,\mathcal{D},k} \mathbf{w}^{j,k}_{\mathrm{CSEM}} \right\|^2_{\mathcal{D}} \\
+ \; & \alpha^{E}_{1,\mathrm{MT}} \sum_{k=1}^{N_{\mathrm{f_{MT}}}} \left\| \Xi^{E,k}_{\mathrm{MT}} \left(\mathbf{f}^{E,k}_{\mathrm{MT}} - G^{E,\mathcal{S},k} \mathbf{w}^{k}_{\mathrm{MT}} \right) \right\|^2_{\mathcal{S}} \\
+ \; & \alpha^{H}_{1,\mathrm{MT}} \sum_{k=1}^{N_{\mathrm{f_{MT}}}} \left\| \Xi^{H,k}_{\mathrm{MT}} \left(\mathbf{f}^{H,k}_{\mathrm{MT}} - G^{H,\mathcal{S},k} \mathbf{w}^{k}_{\mathrm{MT}} \right) \right\|^2_{\mathcal{S}} \\
+ \; & \alpha_{2,\mathrm{MT}} \sum_{k=1}^{N_{\mathrm{f_{MT}}}} \left\| \chi \mathbf{e}^{\mathrm{inc},k}_{\mathrm{MT}} - \mathbf{w}^{k}_{\mathrm{MT}} + \chi G^{E,\mathcal{D},k} \mathbf{w}^{k}_{\mathrm{MT}} \right\|^2_{\mathcal{D}}
\end{aligned}
$$

(first two terms) CSEM data fit

(third term) Lippmann-Schwinger

(fourth and fifth terms) MT data fit

(sixth term) Lippmann-Schwinger

+ Regularization

Contrast-source inversion

foreach iteration

foreach CSEM source and frequency

minimize J w.r.t. $\mathbf{w}_{\mathrm{CSEM}}^{j,k}$

end

foreach MT frequency

minimize J w.r.t. $\mathbf{w}_{\mathrm{MT}}^{k}$

end

Minimize J w.r.t. $\boldsymbol{\chi}$

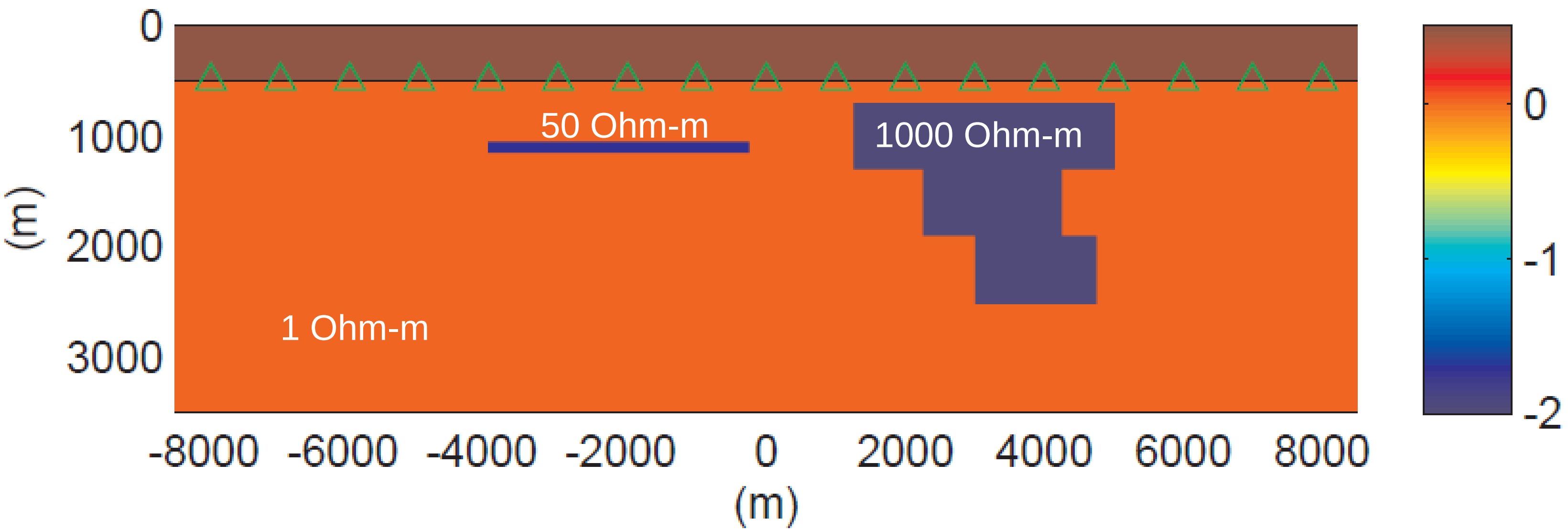
Stop or continue?

end

Objective function and regularization

$$J = F_1\left(\mathcal{W}_{\mathrm{CSEM}}, \mathcal{W}_{\mathrm{MT}}, \boldsymbol{\chi}\right) + \lambda^2 \left\|\boldsymbol{\Omega}\left(\boldsymbol{\chi} - \boldsymbol{\chi}^{\mathrm{ref}}\right)\right\|_{\mathcal{D}}^2$$

Synthetic example - isotropic



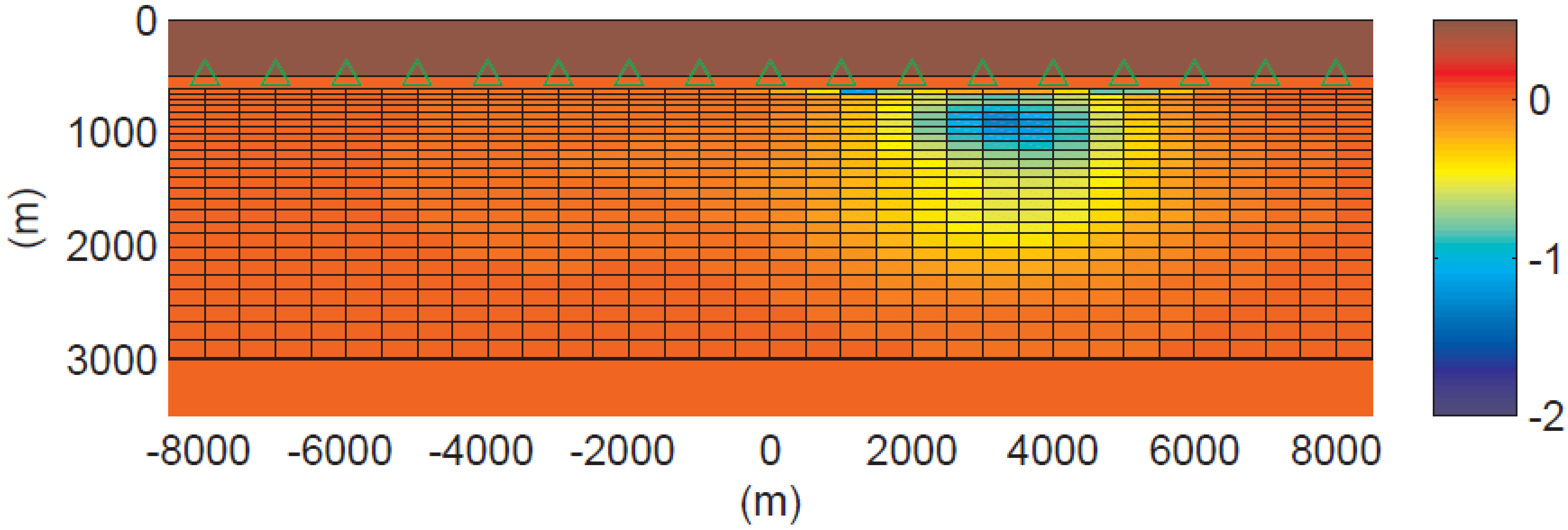
CSEM:

- f=0.25 Hz

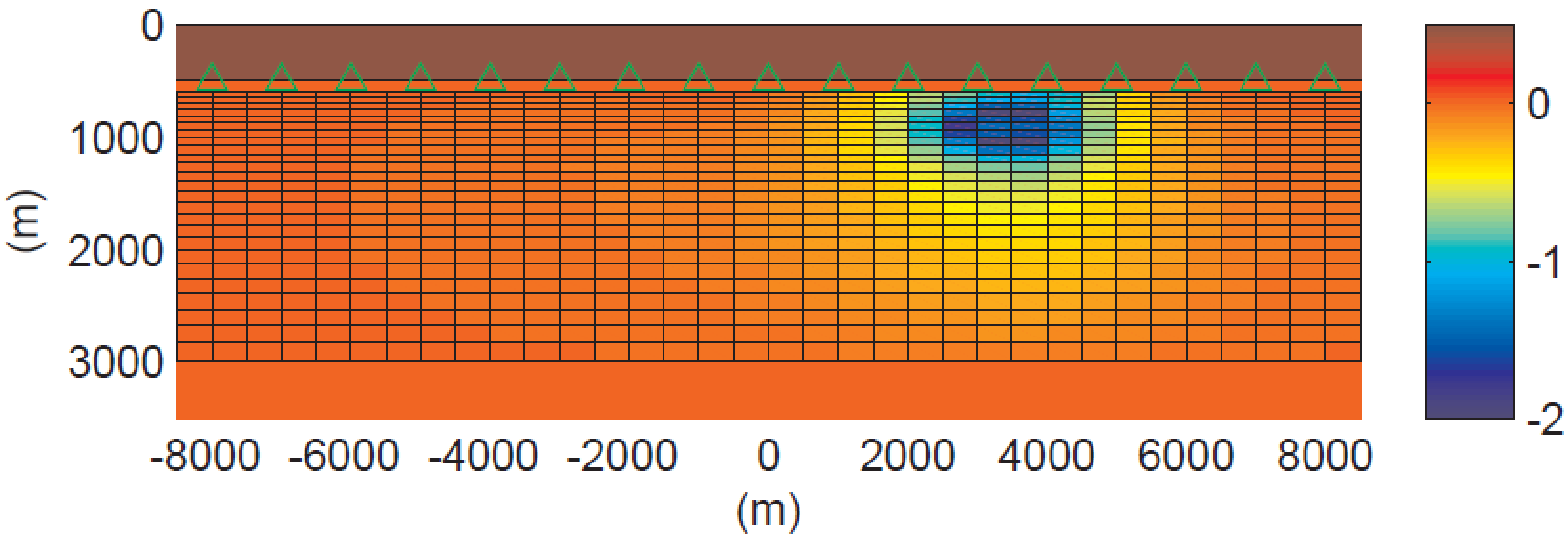
MT:

- T={10, 20, 40, 100} s, f={0.1, 0.05, 0.025, 0.01} Hz

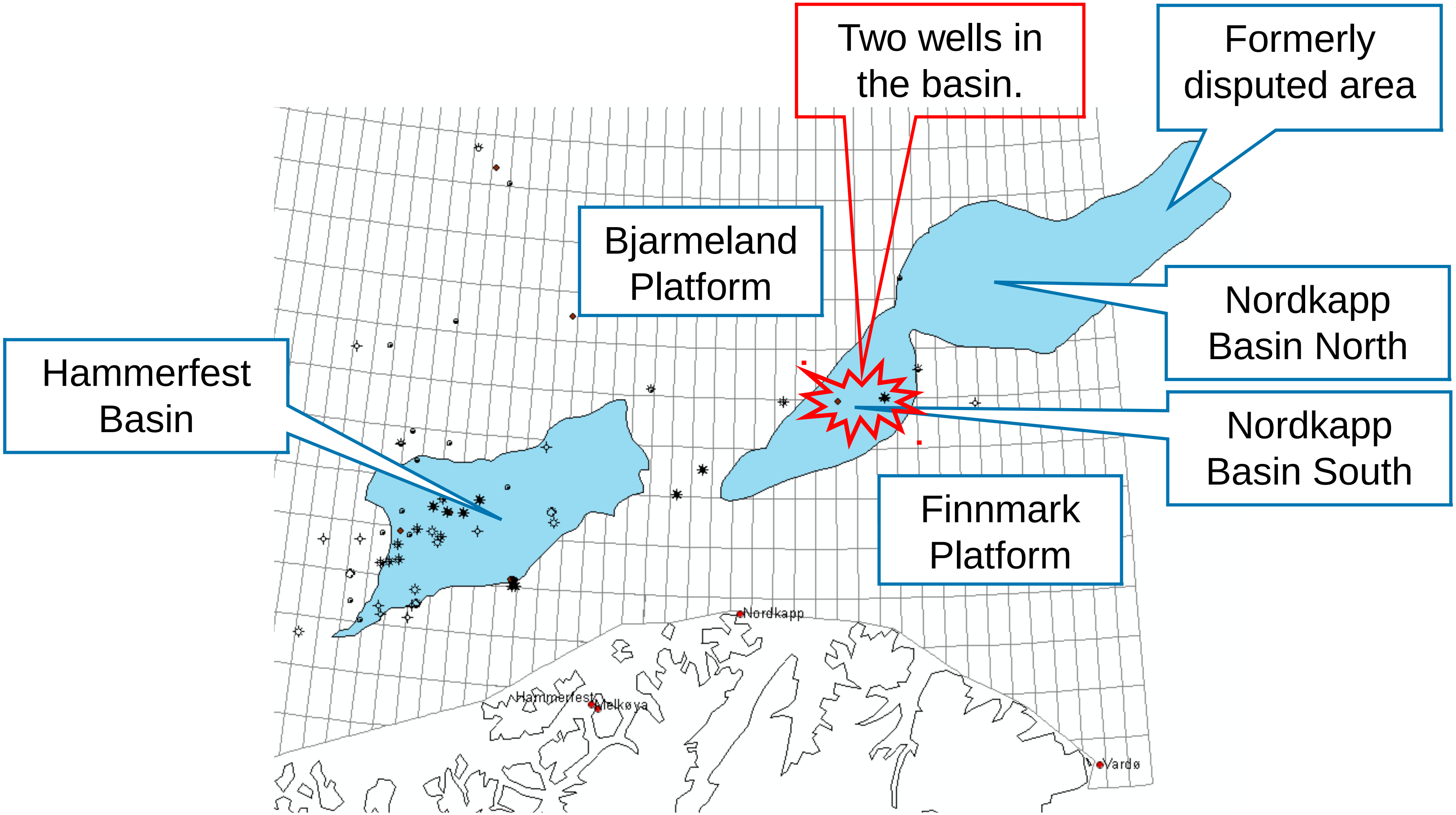
MT inversion



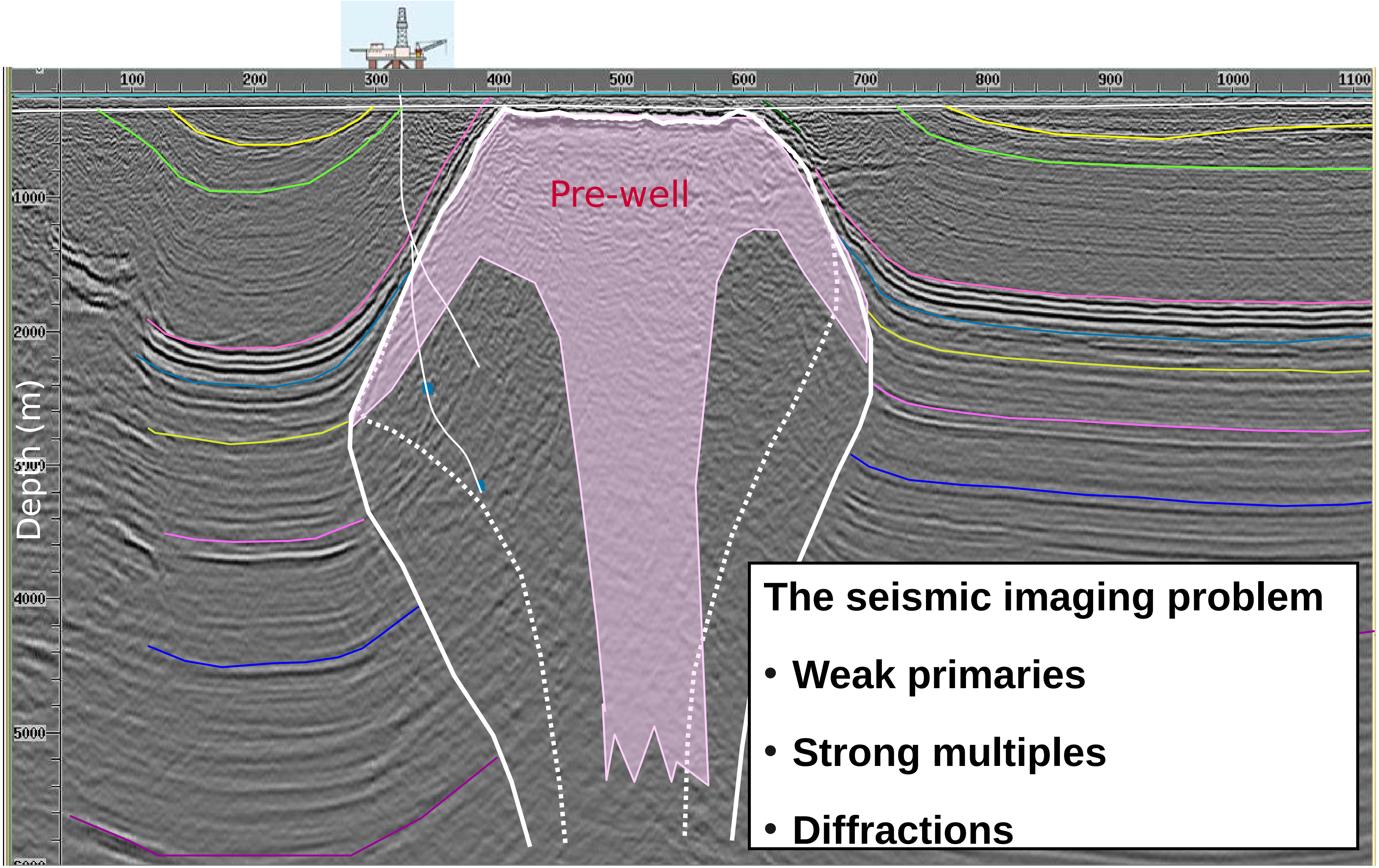
Joint MT+CSEM inversion



The Nordkapp Basin - Location



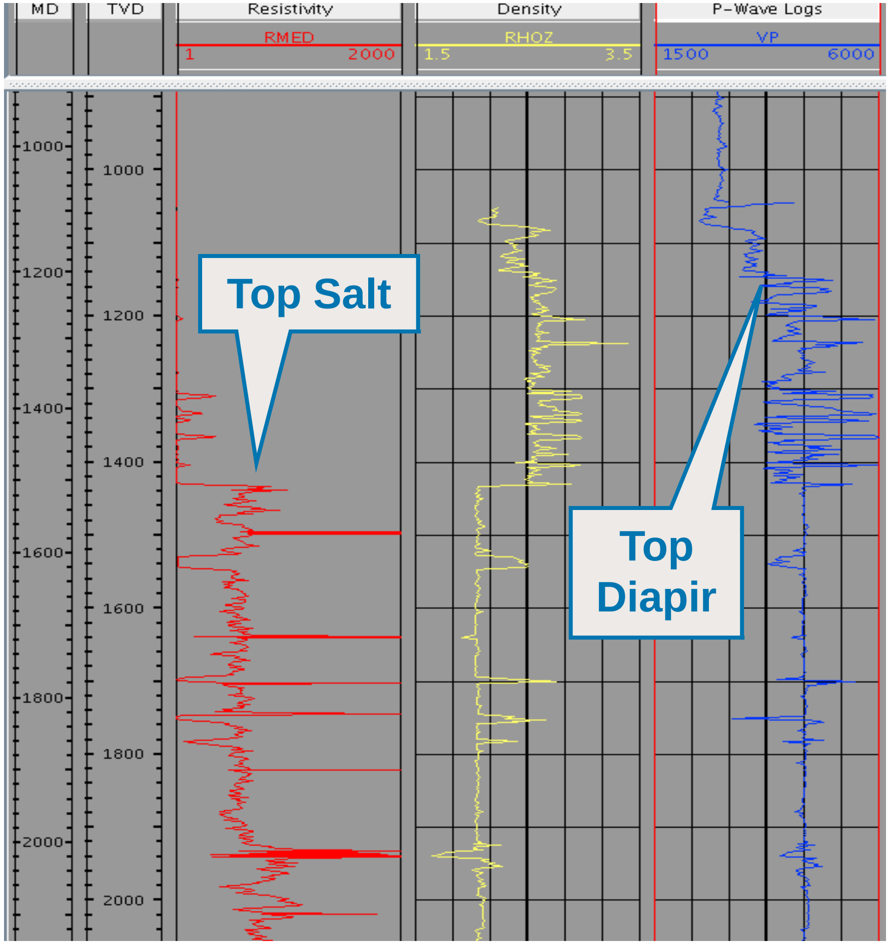
Uranus pre and post-well interpretations



The seismic imaging problem

- **Weak primaries**
- **Strong multiples**
- **Diffractions**

Can we image the salt-sediment interface with non-seismic geophysics?

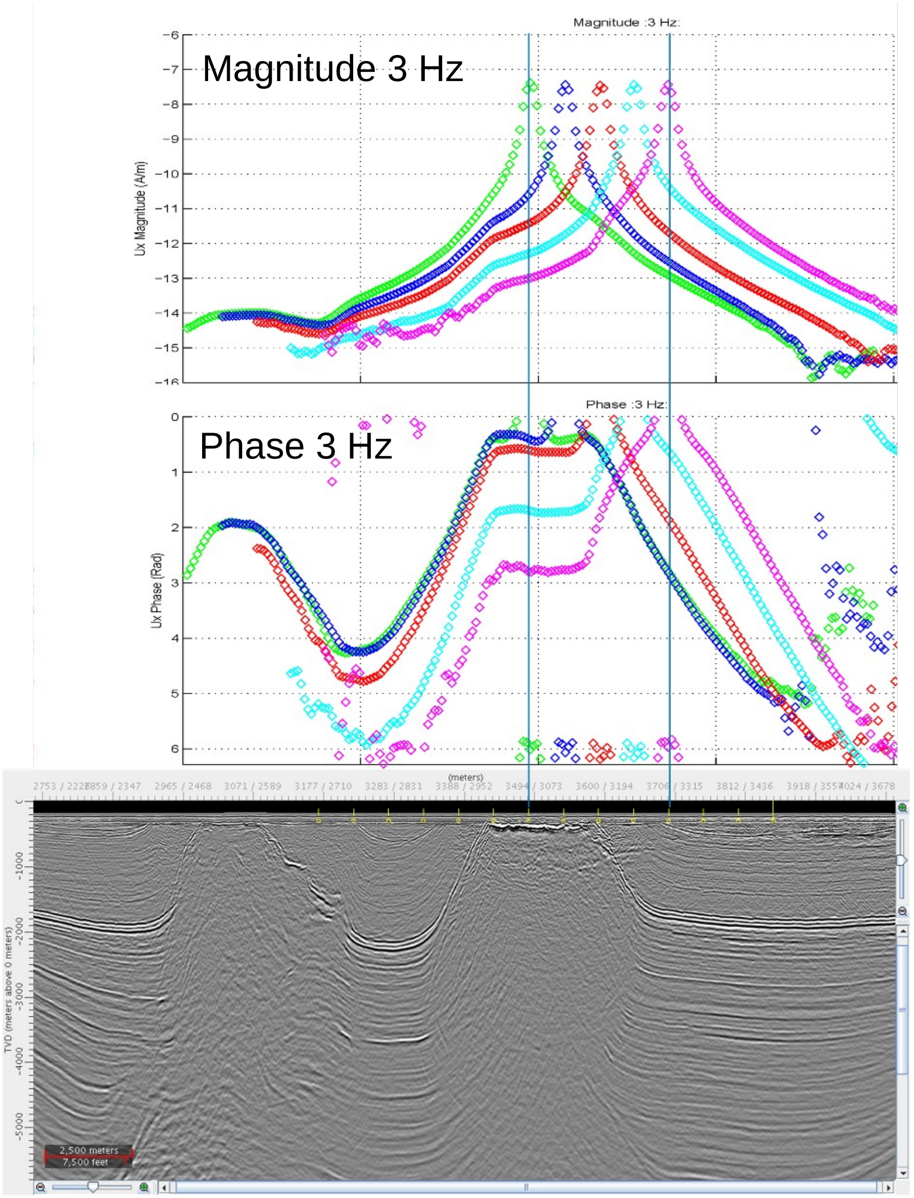


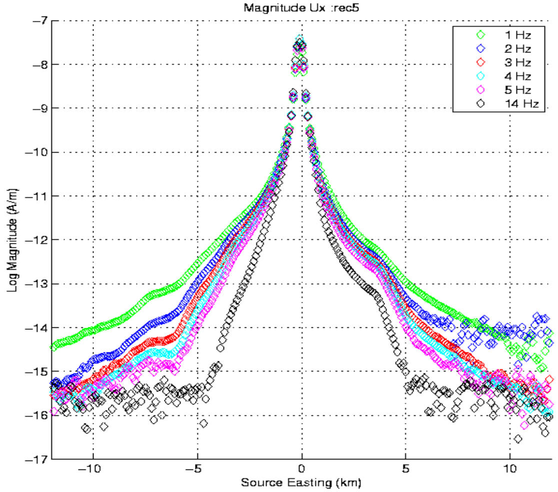
Characteristic properties of rock salt:

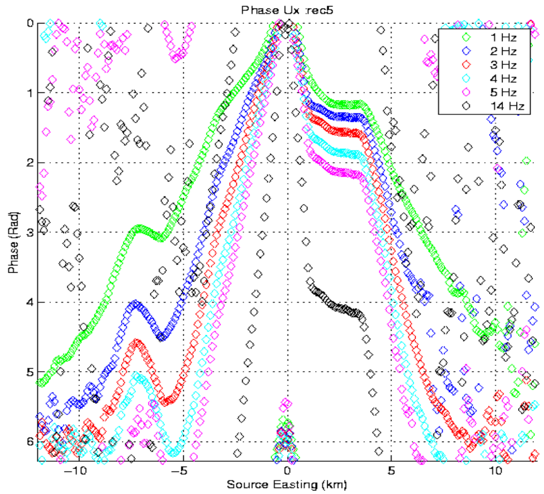
- Low mass density
  - Gravimetry
- High electric resitivity
  - Magnetotellurics
  - Controlled-Source EM
- Seismic is needed to get the details

The failure of the Uranus well triggered major geophysical efforts

CSEM magnitude and phase – inline E field



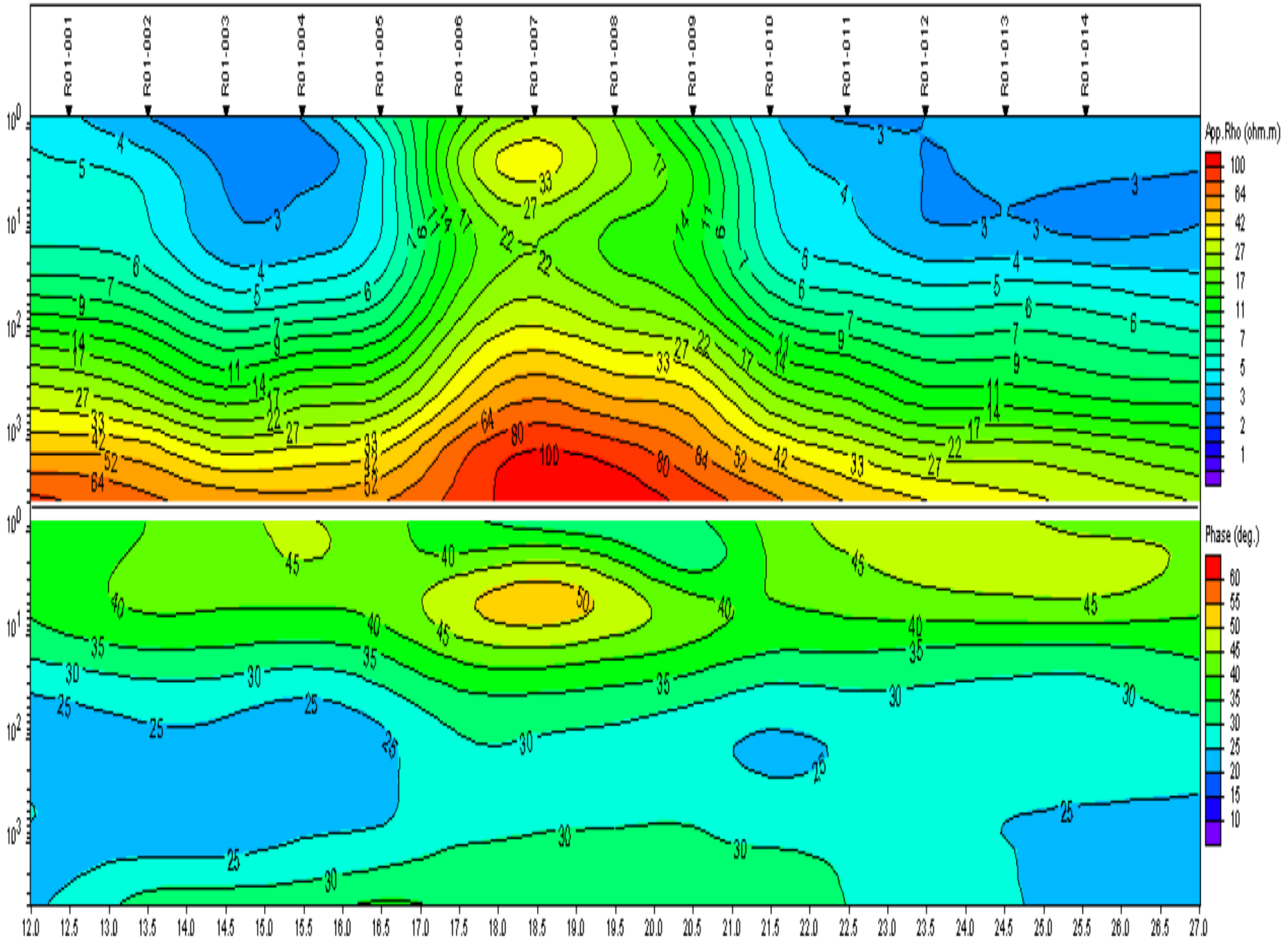




Statoil

MT apparent resistivity

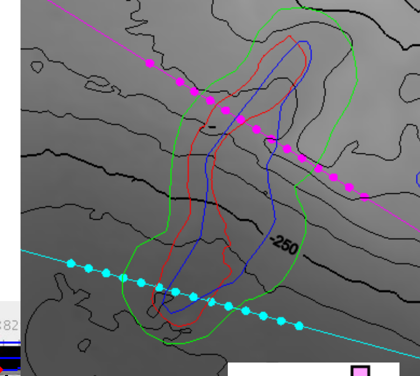
TM mode

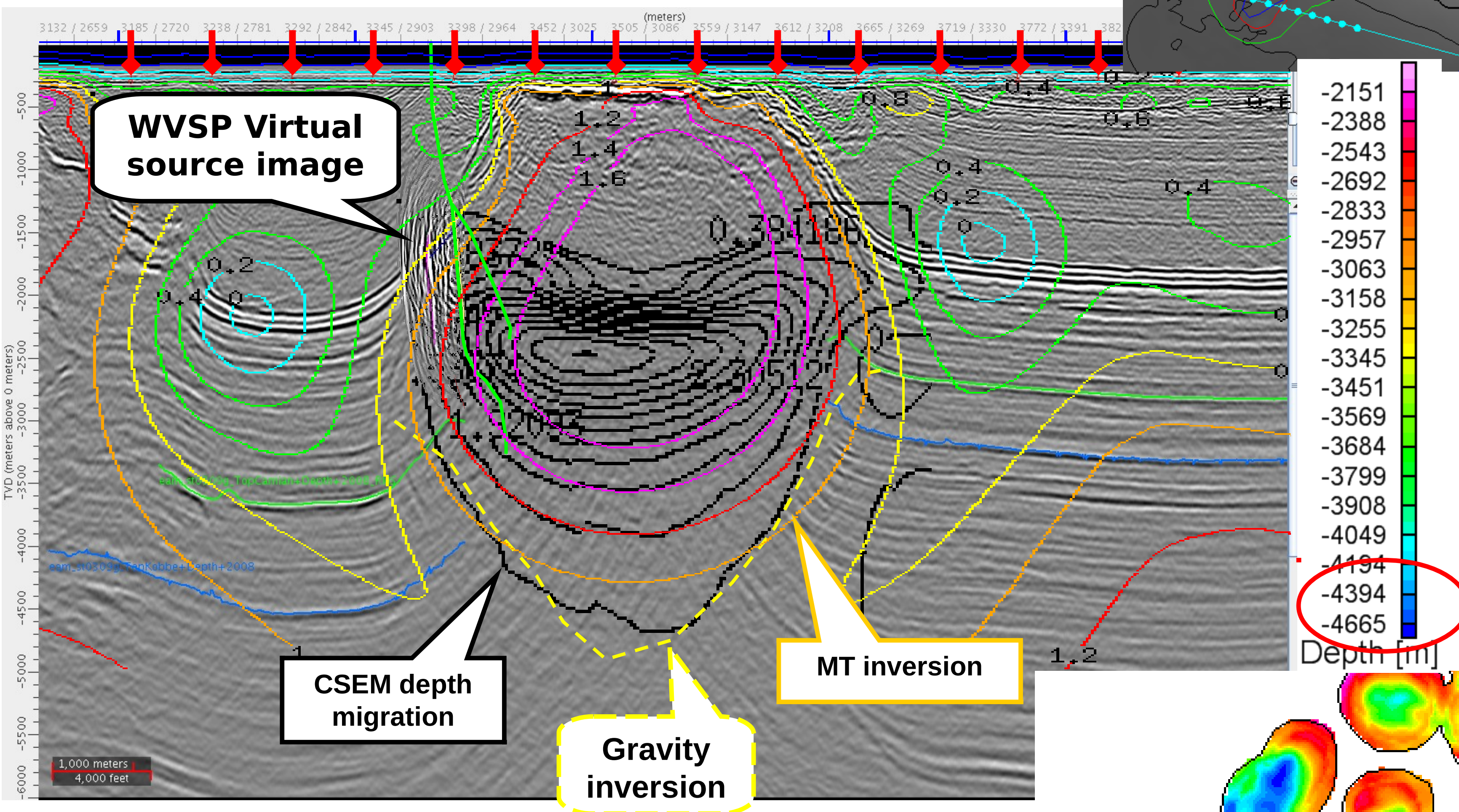


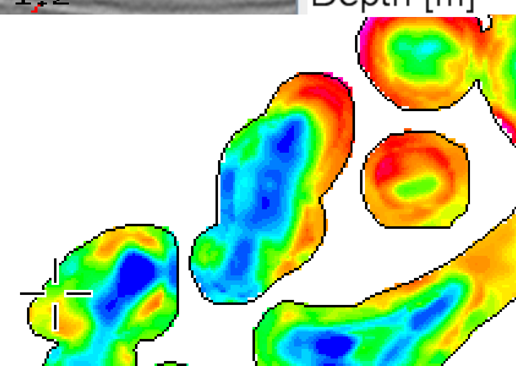
MT processing by Geosystem – Luca Masnaghetti

Statoil

Uranus – previous results

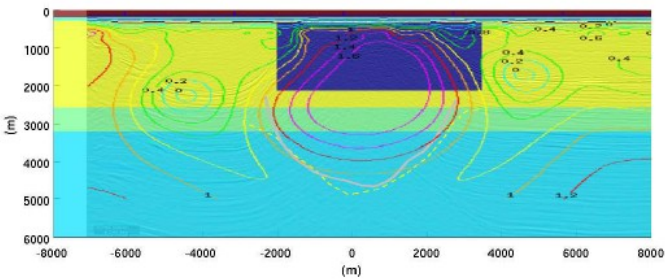




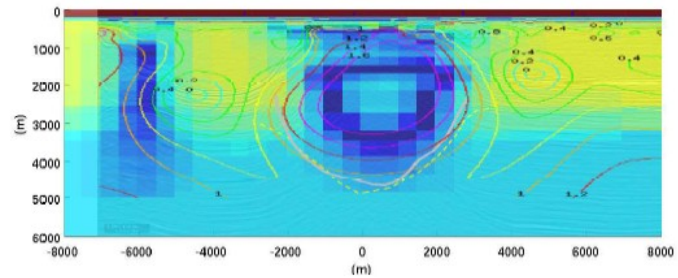


Statoil

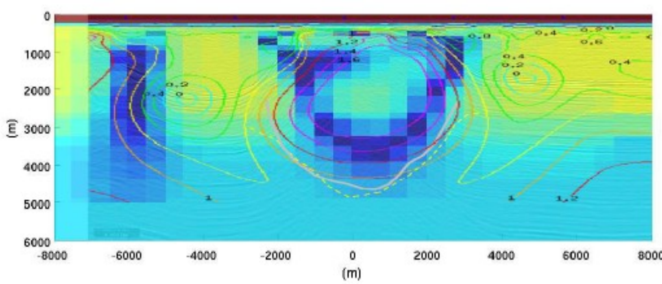
Contrast-source inversion



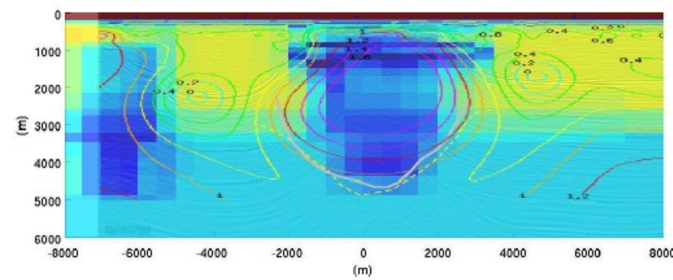
Initial model



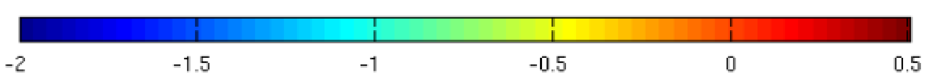
Sequential MT+CSEM inversion



MT inversion



Simultaneous MT+CSEM inversion



Real MT data

Conventional approach

- Impedance tensor Z is standard output from MT processing

$$E_I = Z_{IJ} H_J, \; I, J = 1,2$$

$$\mathbf{E} = \mathbf{ZH}$$

- Inversion of impedance tensor or apparent resistivity

Our approach:

- Return to E- and H-fields

$$\begin{bmatrix} E_x \\ E_y \\ H_x \\ H_y \end{bmatrix} = \begin{bmatrix} 1 & \mathbf{Z} \\ \mathbf{Z}^{-1} & 1 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ H_x \\ H_y \end{bmatrix}$$

- Solve an eigenvalue/eigenvector problem with unit eigenvalues
- Only minor modifications of inversion scheme needed
- Other options being investigated …

Conclusions

- Contrast source inversion developed for diffusive EM fields
- Current implementation in Matlab; large speedup expected by porting to Fortran and cluster
- Tested on real and synthetic CSEM and MT data
- Results on real data from Uranus are in agreement with previous results obtained with other methods