Reverse-time migration velocity analysis

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NTNU – Trondheim
Norwegian University of Science and Technology

Outline

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Introduction

- Automatically obtaining the background velocities for depth migration
- Non-linear optimization problem based on Differential Semblance [Symes and Carazzone, 1991, Shen and Symes, 2008].
- How to obtain a stable RTM Differential Semblance Optimization algorithm

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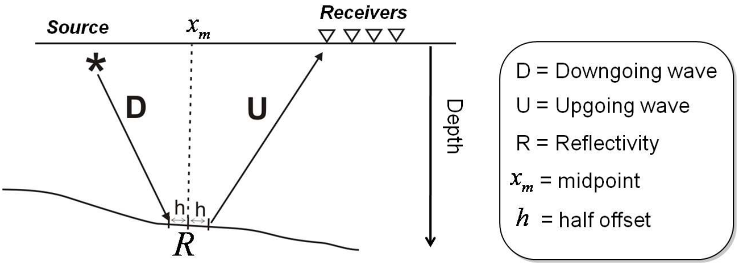
Reverse time migration and velocity analysis

Depth migration = Wavefield extrapolation + crosscorrelation (Claerbout, 1971)



Cross correlation (imaging condition)

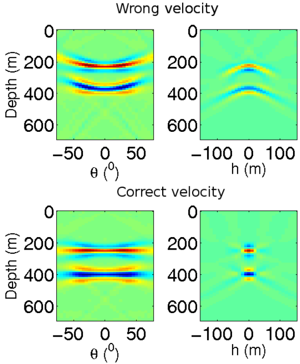
In reverse time migration, R is contructed according to the multi-offset crosscorrelation *imaging condition* [Rickett and Sava, 2002]:

$$R(x,h,z) = \sum_{s} \sum_{t} U(x+h,z,t,s)D(x-h,z,t,s). \qquad (1)$$

Where s represents the source index, t is the time index, D is the forward modeled source wavefield, and U is the reflected wave field, reverse time extrapolated from the receivers.

Common image point gathers (CIPs)

Example of CIPs output by Reverse time migration:



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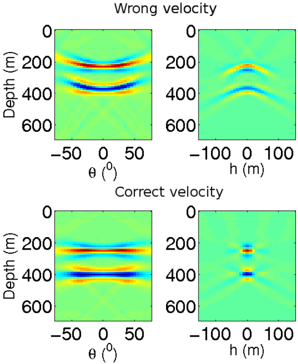

- MVA uses the information on the CIPs to find the correct slowness.

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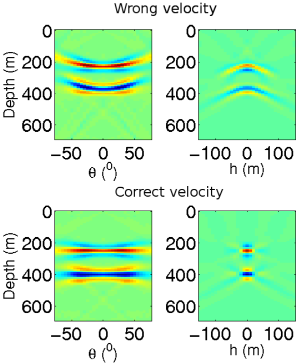

- MVA uses the information on the CIPs to find the correct slowness.
- Improved slowness flattens the angle domain CIPs and focuses the offset domain CIPs.

Differential Semblance

We start by defining our differential semblace misfit function:

$$DS = \frac{1}{2}\|h\partial_z R\|^2 = \frac{1}{2}\int dx \int dh \int dz \; h^2(\partial_z R(x,h,z))^2, \quad (2)$$

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Optimization can be carried out iteratively, at each iteration yielding a new velocity model:

$$c_{k+1}(x,z) = c_k(x,z) - \alpha\nabla_c DS(x,z)$$

Where α is the step length and $\nabla_c DS$ is the gradient of DS with respect to $c(x,z)$.

Gradient Computation

The derivative of equation 2 with respect to velocity $c(x,z)$ can be efficiently computed through the adjoint state method [Chavent, 2009].

$$\begin{aligned}\nabla_c DS(x,z) &= -\sum_s \sum_t \frac{2}{c^3(x,z)} \frac{\partial^2 D}{\partial t^2}(x,z,t,s) D'(x,z,t,s) \\ &\quad -\sum_s \sum_t \frac{2}{c^3(x,z)} \frac{\partial^2 U}{\partial t^2}(x,z,t,s) U'(x,z,t,s)\end{aligned}$$

where $U'(x,z,t,s)$ and $D'(x,z,t,s)$ are adjoint states.

Gradient Computation (2)

Introducing the Green's function:

$$\left(\frac{1}{c^2(x,z)}\frac{\partial^2}{\partial t^2} + \nabla^2\right) g(x,z,t;x',z',t') = \delta(x-x')\delta(z-z')\delta(t-t'),$$

D' and U' are then found to be the solutions of two simulations:

$$D'(x,z,t,s) = \int dx' \int dz' \; g(x,z,0;x',z',t) * \left(\int dh \; h^2 \partial_z^2 R(x'+h,h,z')U(x'+2h,z',t,s)\right),$$

$$U'(x,z,t,s) = \int dx' \int dz' \; g(x,z,t;x',z',0) * \left(\int dh \; h^2 \partial_z^2 R(x'-h,h,z')D(x'-2h,z',t,s)\right),$$

where $*$ denotes time convolution.

Gradient Computation (3)

A step by step procedure to compute the gradient follows:

1. Construct R, and at the same time store the direct states U and D for each shot.

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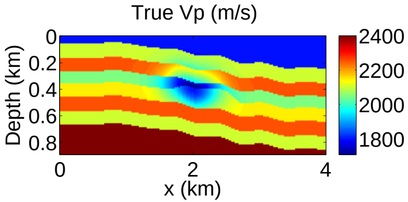
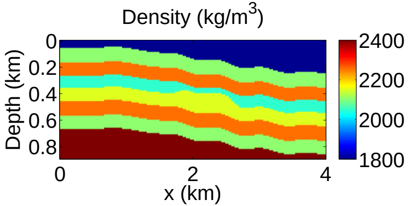
2. Perform the two simulations for each shot to compute the adjoint states D' and U', and at each time step crosscorrelate, respectively, with the D and U to build the source and receiver parts of the gradient.
3. Stack the source and receiver parts of the gradient over all shots to obtain the full gradient.

2D Shallow lens model





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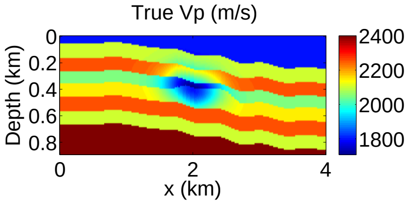
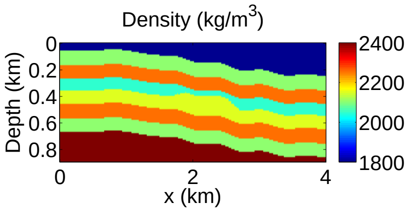
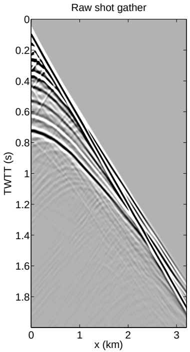
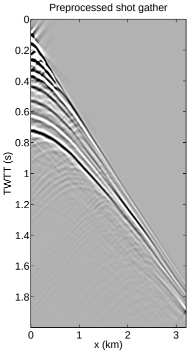
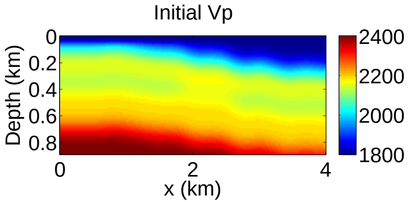


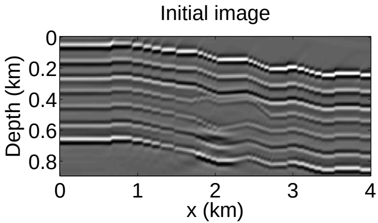


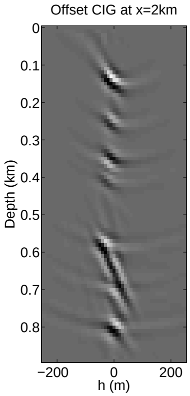


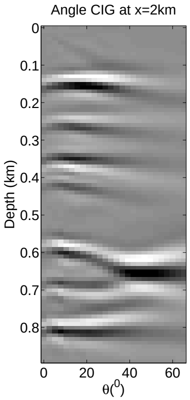


Migration

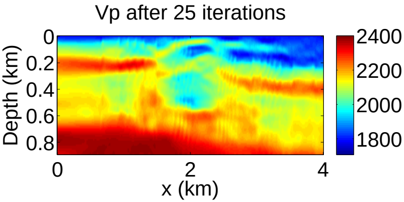

Initial Vp
Depth (km)
0
0.2
0.4
0.6
0.8
0
2
4
x (km)
2400
2200
2000
1800

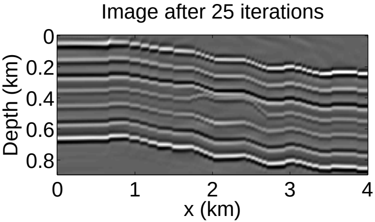

Initial image
Depth (km)
0
0.2
0.4
0.6
0.8
0
1
2
3
4
x (km)

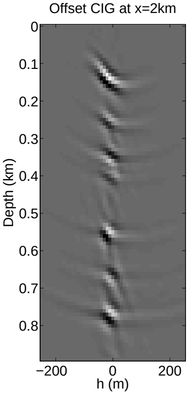

Offset CIG at x=2km
Depth (km)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
−200
0
200
h (m)

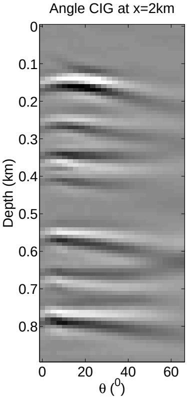

Angle CIG at x=2km
Depth (km)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0
20
40
60
θ(0)

Optimization









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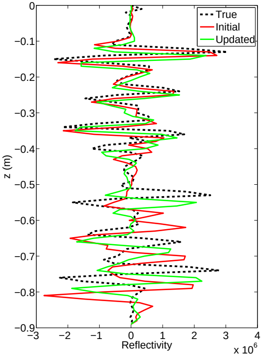
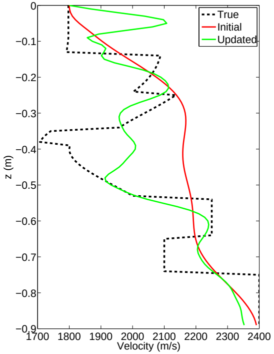
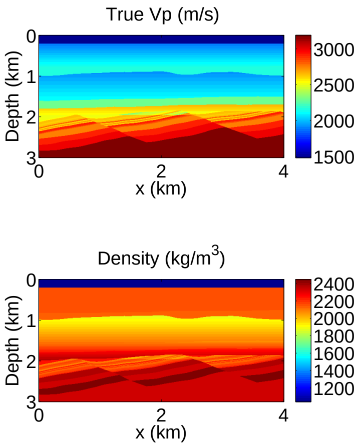




2D Gullfaks model



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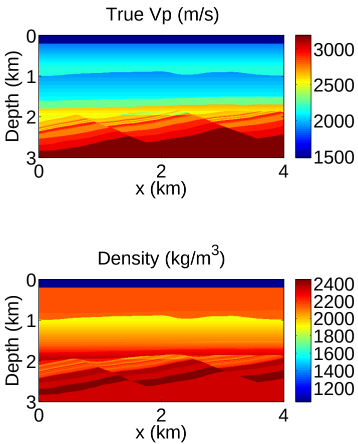

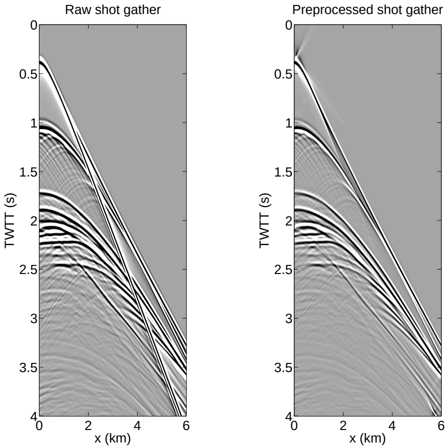

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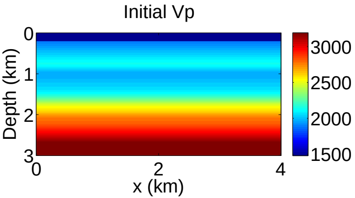
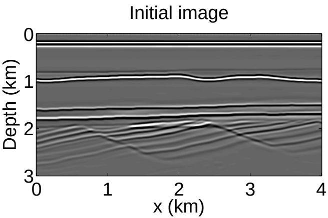
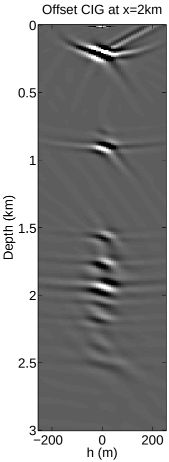
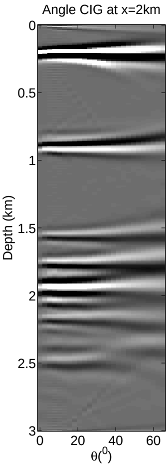
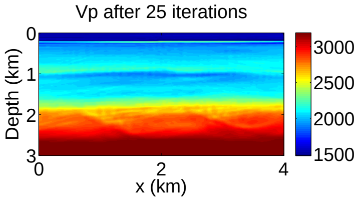

Initial Vp
Depth (km)
0
1
2
3
0
2
4
x (km)
3000
2500
2000
1500

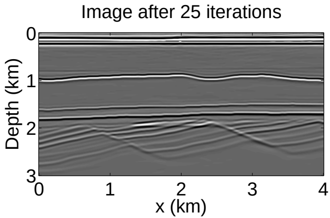

Initial image
Depth (km)
0
1
2
3
0
1
2
3
4
x (km)

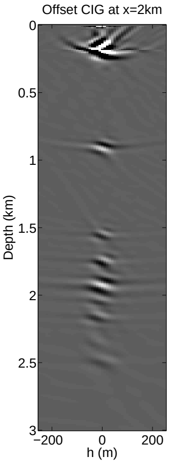

Offset CIG at x=2km
Depth (km)
0
0.5
1
1.5
2
2.5
3
−200
0
200
h (m)

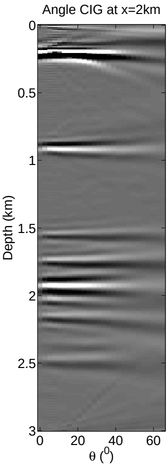

Angle CIG at x=2km
Depth (km)
0
0.5
1
1.5
2
2.5
3
0
20
40
60
θ(0)

Optimization

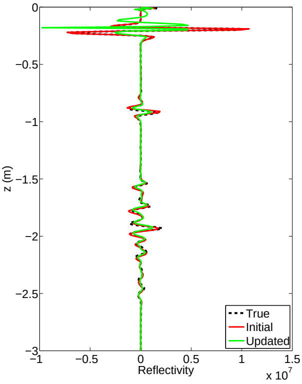


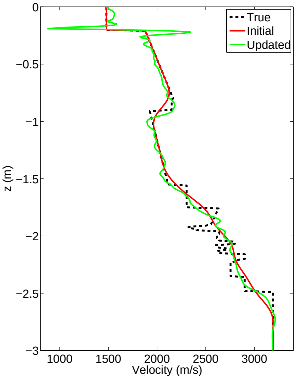


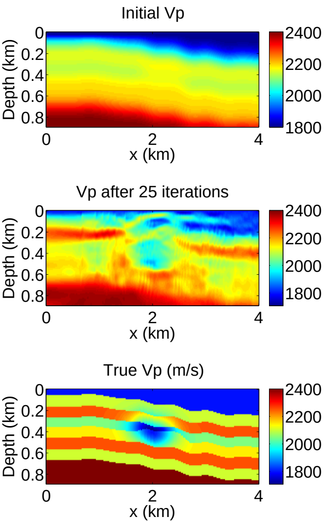


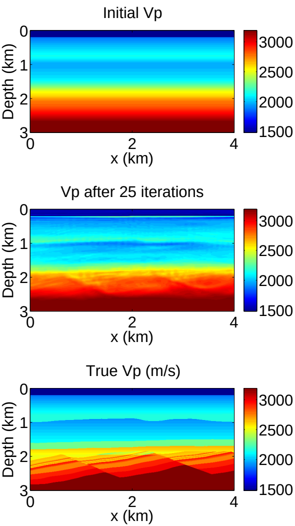


Optimization






Initial Vp
Depth (km)
0
0.2
0.4
0.6
0.8
0
2
4
x (km)
2400
2200
2000
1800
Vp after 25 iterations
Depth (km)
0
0.2
0.4
0.6
0.8
0
2
4
x (km)
2400
2200
2000
1800
True Vp (m/s)
Depth (km)
0
0.2
0.4
0.6
0.8
0
2
4
x (km)
2400
2200
2000
1800


Initial Vp
Depth (km)
0
1
2
3
0
2
4
x (km)
3000
2500
2000
1500
Vp after 25 iterations
Depth (km)
0
1
2
3
0
2
4
x (km)
3000
2500
2000
1500
True Vp (m/s)
Depth (km)
0
1
2
3
0
2
4
x (km)
3000
2500
2000
1500

Acknowledgements

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