RTM in VTI media using the Rapid Expansion Method (REM)

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Outline

- Motivation
- Wave equation for acoustic VTI media
  - Dispersion relation for 3D acoustic VTI media
  - Du et al. (2008) coupled system of second-order PDEs
  - Faqi Liu's (2009) P- and SV-wave equations
  - A new approach to separate P- and SV wave components
- Rapid expansion method - REM
- Numerical results - Impulse response and VTI Hess dataset
- Conclusions
- Acknowledgments

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Motivation

- The importance of anisotropy has been identified several decades ago. Indeed, ignoring the effect of anisotropy in imaging may results in significant mispositioning of steeply structures.
- Conventional isotropic methods for seismic data processing are subject to errors in transversely isotropic (TI) media.
- Reverse time migration (RTM) is a depth migration algorithm. By using the full wave equation, RTM implicitly includes multiple arrival paths and has no dip limitation, enabling the imaging of complex structures.

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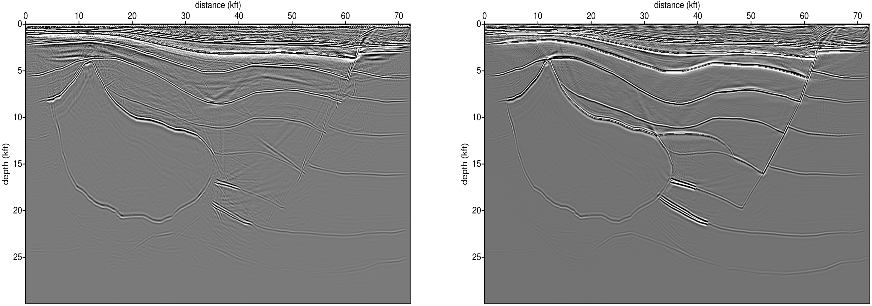
Migration results for a VTI dataset



Isotropic RTM (left) and anisotropic RTM (right) using the correct model parameters (left).

Wave equation in acoustic VTI media

Acoustic anisotropy is introduced by setting the shear wave velocity to zero, i.e., $v_s = 0$, along the symmetric axis (Alkhalifah, 1998).

The Dispersion relation for waves in 3D acoustic VTI media (Alkhalifah, 2000) is given by:

$$\omega^4 - \left[v_h^2\, k_r^2 + v_{po}^2 k_z^2\right] \omega^2 - v_{po}^2 (v_n^2 - v_h^2)\, k_r^2\, k_z^2 = 0 \qquad (1)$$

- k_x, k_z and k_z are wavenumbers in the x, y and z directions, $k_r^2 = k_x^2 + k_y^2$
- ω is the angular frequency; v_{po} is the vertical P velocity.
- $v_n = v_{po}\sqrt{1 + 2\delta}$ is the P-wave normal moveout (NMO) velocity;
- $v_h = v_{po}\sqrt{1 + 2\epsilon}$ is the horizontal P velocity;
- δ and ϵ are anisotropic parameters - Thomsen (1986).

The equivalent partial differential equation (PDE) follows immediately as

$$\begin{aligned}\frac{\partial^4 q}{\partial t^4} &= v_h^2\left(\frac{\partial^4 q}{\partial x^2\partial t^2}+\frac{\partial^4 q}{\partial y^2\partial t^2}\right)+v_{po}^2\left(\frac{\partial^4 q}{\partial z^2\partial t^2}\right) \\ &+ v_{po}^2(v_n^2-v_h^2)\left(\frac{\partial^4 q}{\partial x^2\partial z^2}+\frac{\partial^4 q}{\partial y^2\partial z^2}\right)\end{aligned} \qquad (2)$$

which is problematic to solve as it is a fourth-order equation in time.

Alkhalifah (2000) derived the following coupled system of second-order PDEs for the wavefield p(x,y,z,t) and the auxiliary wavefield function q(x,y,z,t):

$$
\begin{aligned}
\frac{\partial^2 p}{\partial t^2} &= v_h^2 \left(\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} \right) + v_{po}^2 \left(\frac{\partial^2 p}{\partial z^2} \right) \\
&+ v_{po}^2 (v_n^2 - v_h^2) \left(\frac{\partial^4 q}{\partial x^2 \partial z^2} + \frac{\partial^4 q}{\partial y^2 \partial z^2} \right) \qquad (3) \\
\frac{\partial^2 q}{\partial t^2} &= p
\end{aligned}
$$

Zhou et al. (2006) using a different auxiliary function q, derived the following coupled system of equations for wavefield p and auxiliary q wavefield,

$$
\begin{aligned}
\frac{\partial^2 p}{\partial t^2} &= v_n^2 \left(\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 q}{\partial x^2} + \frac{\partial^2 q}{\partial y^2} \right) + v_{po}^2 \frac{\partial^2 p}{\partial z^2} \\
\frac{\partial^2 q}{\partial t^2} &= (v_h^2 - v_n^2) \left(\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 q}{\partial x^2} + \frac{\partial^2 q}{\partial y^2} \right)
\end{aligned} \tag{4}
$$

Compared with 3, Zhou's equations use lower order derivatives and are more convenient for computational efficiency.

Wave equation - Du et at. (2008)

Introducing the new auxiliary function

$$q(\omega, k_x, k_y, k_z) = \frac{\omega^2 + (v_n^2 - v_h^2)\left(k_x^2 + k_y^2\right)}{\omega^2}\, p(\omega, k_x, k_y, k_z) \quad (5)$$

Now the equation 1 can be written as

$$\begin{aligned} \omega^2 p(\omega, k_x, k_y, k_z) &= v_h^2\left(k_x^2 + k_y^2\right) p(\omega, k_x, k_y, k_z) \quad (6) \\ &+ v_{po}^2 k_z^2\, q(\omega, k_x, k_y, k_z) \end{aligned}$$

Wave equation - Du et at. (2008)

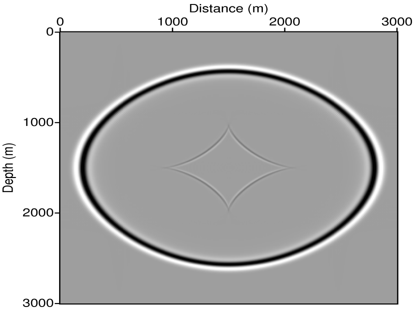
Applying an inverse Fourier to both sides of the previous two equations, we obtain the following pseudo-acoustic VTI system of equations

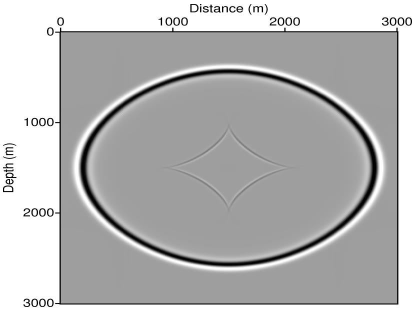
$$\begin{aligned} \frac{\partial^2 p}{\partial t^2} &= v_h^2 \left(\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} \right) + v_{po}^2 \frac{\partial^2 q}{\partial z^2} \\ \frac{\partial^2 q}{\partial t^2} &= v_n^2 \left(\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} \right) + v_{po}^2 \frac{\partial^2 q}{\partial z^2} \end{aligned} \tag{7}$$

Or using the following matrix formulation (2D case):

$$\frac{\partial^2}{\partial t^2} \begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} v_x^2 \frac{\partial^2}{\partial x^2} & v_{po}^2 \frac{\partial^2}{\partial z^2} \\ v_n^2 \frac{\partial^2}{\partial x^2} & v_{po}^2 \frac{\partial^2}{\partial z^2} \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix} \tag{8}$$

Wavefield snapshot in a homogeneous VTI medium





Impulse response computed using Du's equations solved by REM. Homogeneous VTI medium vith: $v_z = 3000m/s$, $\epsilon = 0.24$ and $\delta = 0.1$. p-wavefield (left) and q-wavefield (right).

Decoupled wave equations for P and SV waves - VTI media

Recently, Liu et al. (2009) factorized the dispersion relation presented by Alkhalifah (2000) and obtain two separate P- and SV-wave dispersion relations to

$$\omega^2 = \frac{1}{2}\left[v_h^2\, k_r^2 + v_{po}^2 k_z^2\right] \pm \frac{1}{2}\left[v_h^2\, k_r^2 + v_{po}^2 k_z^2\right] \left[1 + \frac{4v_{po}^2(v_n^2 - v_h^2)\, k_r^2\, k_z^2}{\left[v_h^2\, k_r^2 + v_{po}^2 k_z^2\right]^2}\right]^{1/2} \tag{9}$$

We expand the square root to first order ($\sqrt{1+X} = 1 + \frac{1}{2}X$) and obtain

Decoupled wave equations for P and SV waves - VTI media

For P-Wave

$$\omega^2 = v_{po}^2 k_z^2 + v_h^2\, k_r^2 + \frac{(v_n^2 - v_h^2)\, k_r^2\, k_z^2}{k_z^2 + F\, k_h^2} \tag{10}$$

and

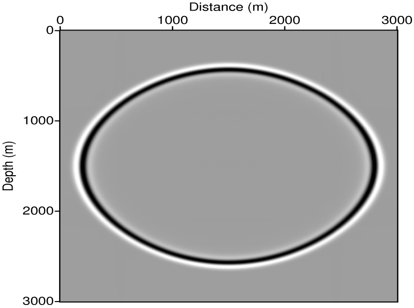
For SV-wave

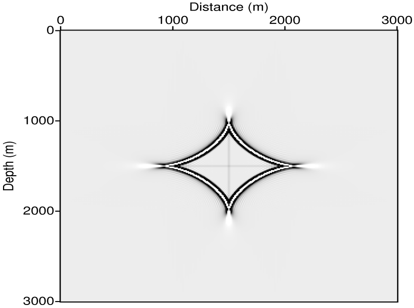
$$\omega^2 = -\frac{(v_n^2 - v_h^2)\, k_r^2\, k_z^2}{k_z^2 + F\, k_r^2} \tag{11}$$

where, here, $F = \frac{v_h^2}{v_{po}^2} = 1 + 2\epsilon$

For the equation for the SV-wave to be stable we must have that $v_h^2 - v_n^2 \geq 0$ or $\epsilon \geq \delta$. It does not, however, represent realistic SV-wave propagation.

Wavefield snapshotin a homogeneous VTI medium





P-wave wavefield (left) and SV-wave wavefield (right) from decoupled P- and SV-wave equations proposed by Liu et at. (2009) also solved by REM.

We start with the exact dispersion relations for VTI media as derived by Tsvankin(1996):

$$\frac{v^2(\theta)}{v_{po}^2} = 1+\epsilon\sin^2\theta-\frac{f}{2}\pm\frac{f}{2}\left[1+\frac{2\epsilon\sin^2\theta}{f}\right]\left[1-\frac{2(\epsilon-\delta)\sin^2 2\theta}{f(1+\frac{2\epsilon\sin^2\theta}{f})^2}\right]^{1/2} \tag{12}$$

where θ is the phase angle measured from the symmetry axis. The plus sign corresponds to the P-wave and the minus sign corresponds to the SV-wave.

Here

$$f = 1-\left(\frac{v_{so}}{v_{po}}\right)^2 \tag{13}$$

v_{po} and v_{so} are P- and S-wave velocities respectively, and ϵ and δ are the Thomsen (1986) parameters.

Approximate scalar wave equations in VTI media

We expand the square root to first order ($\sqrt{1+X} = 1 + \frac{1}{2}X$) and obtain the approximation

P-wave

$$\frac{v^2(\theta)}{v_{po}^2} = 1 + 2\epsilon \sin^2\theta - \frac{(\epsilon-\delta)\sin^2 2\theta}{2(1+\frac{2\epsilon\sin^2\theta}{f})} \tag{14}$$

and

SV-wave

$$\frac{v^2(\theta)}{v_{po}^2} = 1 - f + \frac{(\epsilon-\delta)\sin^2 2\theta}{2(1+\frac{2\epsilon\sin^2\theta}{f})} \tag{15}$$

In order to develop these equation further we introduce

$$\begin{aligned} v_h^2 &= v_{po}^2(1+2\epsilon) \\ v_n^2 &= v_{po}^2(1+2\delta). \end{aligned} \tag{16}$$

With $\sin(\theta) = \frac{v(\theta) k_r}{\omega}$ and $\cos(\theta) = \frac{v(\theta) k_z}{\omega}$ and

$$v^2(\theta) = \frac{\omega^2}{k_r^2 + k_z^2} \tag{17}$$

The results are the dispersion relations

P-wave

$$\omega^2 = v_{po}^2 k_z^2 + v_h^2\, k_r^2 - \frac{(v_h^2 - v_n^2)\, k_r^2\, k_z^2}{k_z^2 + F\, k_r^2} \tag{18}$$

and

SV-wave

$$\omega^2 = v_{so}^2 (k_r^2 + k_z^2) + \frac{(v_h^2 - v_n^2)\, k_r^2\, k_z^2}{k_z^2 + F\, k_r^2} \tag{19}$$

Here

$$F = 1 + \frac{2\epsilon}{f} = \frac{v_h^2 - v_{so}^2}{v_{po}^2 - v_{so}^2} \tag{20}$$

The new equations 18 and 19 are good approximations for the P- and SV-wave dispersion relation if

$$\left| \frac{2\,(\epsilon - \delta)\,\sin^2 2\theta}{f(1 + \frac{2\,\epsilon\,\sin^2\theta}{f})^2} \right| << 1 \tag{21}$$

Approximate scalar wave equations in VTI media

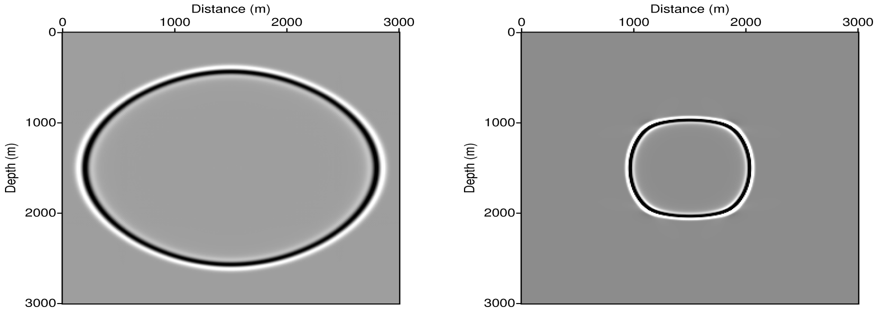
When we set $v_{so} = 0$ (or $f = 1$) equations 18 and 19 reduce to the equations 10 and 11, as derived from Alkhalifah (2000).

If we further set $\epsilon = 0$ in this expression, then $F = 1$, and equation 18 reduces to

$$\omega^2 = v_{po}^2 k_z^2 + v_h^2\, k_r^2 - \frac{(v_h^2 - v_n^2)\, k_r^2\, k_z^2}{k_z^2 + k_r^2} \qquad (22)$$

which is the dispersion relation used by Etgen and Brandsberg-Dahl (2009) and Crawley et al. (2010).

Wavefield snapshot in a homogeneous VTI medium



P-wave wavefield (left) and SV-wave wavefield (right) from decoupled P- and SV-wave equations proposed here, by the REM solving equations 18 and 19.

Pure P-wave equation - Implementation

Based on the work of Zhang et al. (2009), the two-way wave equation can be transformed to a first order in time that is given by:

$$\left(\frac{\partial}{\partial t} + i\Phi\right) P(x, y, z, t) = 0 \tag{23}$$

where P is the complex pressure wavefield and Φ is a pseudo-differential operator in the space domain.

In isotropic media, it is defined by $\Phi = v\sqrt{-\nabla^2}$ or by its symbol $\varphi = v(x, y, z)\sqrt{k_x^2 + k_y^2 + k_z^2}$ where v is the velocity in space domain.

Pure P-wave equation - Implementation

To produce anisotropic wave propagation, without adding spurious waves, we can use the expression 18 and in this case we have:

$$\varphi = \sqrt{v_{po}^2 k_z^2 + v_h^2\, k_r^2 - \frac{\left(v_h^2 - v_n^2\right) k_r^2\, k_z^2}{k_z^2 + F\, k_r^2}} \tag{24}$$

The solution of equation 23 is given by:

$$P(t + \Delta t) = e^{-i\Phi\Delta t}\, P(t) \tag{25}$$

Adding $P(t - \Delta t) = e^{i\Phi\Delta t}\, P(t)$ to equation 25 we obtain

$$p(t + \Delta t) + p(t - \Delta t) = 2\cos(\Phi\Delta t) p(t) \tag{26}$$

Now we can revert to p as the imaginary part is decoupled (cosine is real) - Equation 26 can be now evaluated by the rapid expansion method (REM) (Pestana and Stoffa, 2010).

The temporal procedure to solving equation 8 using the rapid expansion method (REM) (Pestana and Stoffa, 2010) can be expressed symbolically as

$$\vec{u}^{n+1} + \vec{u}^{n-1} = 2\ cos(A\Delta t)\vec{u}^{n} \tag{27}$$

where,

$$\vec{u}^{n} = (p, q)^{T}$$

and

$$A = \begin{pmatrix} v_x^2 \frac{\partial^2}{\partial x^2} & v_{po}^2 \frac{\partial^2}{\partial^2 z} \\ \\ v_n^2 \frac{\partial^2}{\partial x^2} & v_{po}^2 \frac{\partial^2}{\partial^2 z} \end{pmatrix}$$

Time evolution - Rapid expansion method (REM)

The cosine function is given by (Kosloff et. al, 1989)

$$\cos(A\,\Delta t) = \sum_{k=0}^{M} C_{2k}\, J_{2k}(R\,\Delta t)\, Q_{2k}\left(\frac{iA}{R}\right) \tag{28}$$

Chebyshev polynomials recursion is given by:

$$Q_{k+2}(w) = (4w^2 + 2)\, Q_k(w) \; - \; Q_{k-2}\,(w)$$

with the initial values: $Q_0(w) = 1$ and $Q_2(w) = 1 + 2w^2$

For 2D case: $R = \pi v_{max} \sqrt{\frac{1}{\Delta x^2} + \frac{1}{\Delta z^2}}$,

The summation can be safely truncated with a $M > R\,\Delta t$ (Tal-Ezer, 1987).

Spatial derivatives

- Fourier method:

$$\frac{\partial^2 P}{\partial x^2} = IFFT[-{k_x}^2 FFT[P(x)]]$$

- Finite difference:

$$\frac{\partial^2 P_j^n}{\partial x^2} \approx \frac{\delta^2 P_j^n}{\delta x^2} = \frac{1}{\Delta x^2} \sum_{l=-N}^{N} C_l \, P_{j+l}^n$$

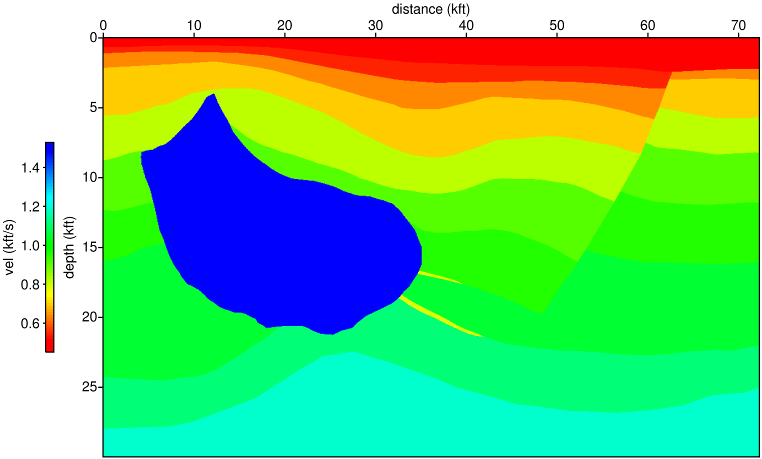
- Convolutional filter (FIR):
  2nd order derivative on regular grids is replaced with a convolutional Finite Impulse Response filter

$$FIR(l) = D_2(l) * H(l)$$

where $H(l)$ is a Hanning taper.

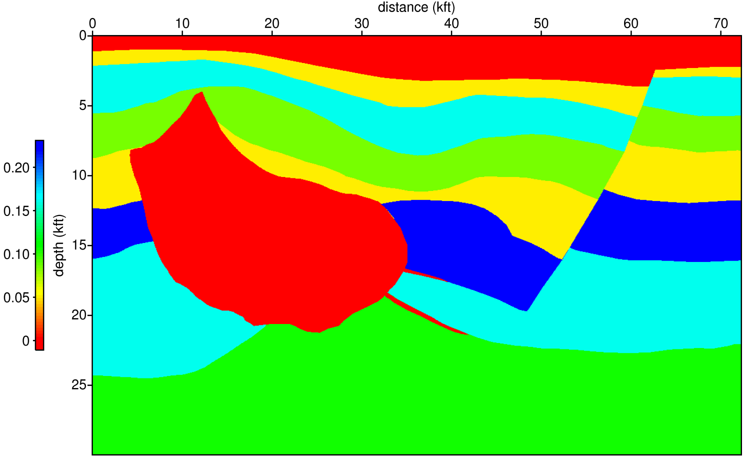
Hess VTI model

Velocity model



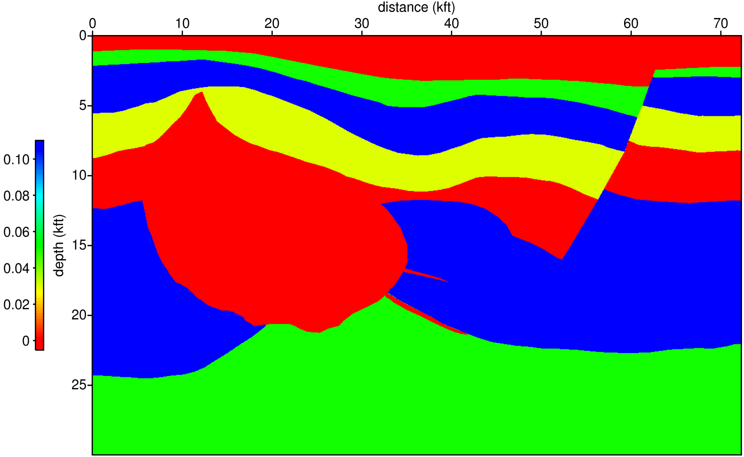
Hess VTI model

Epsilon parameter

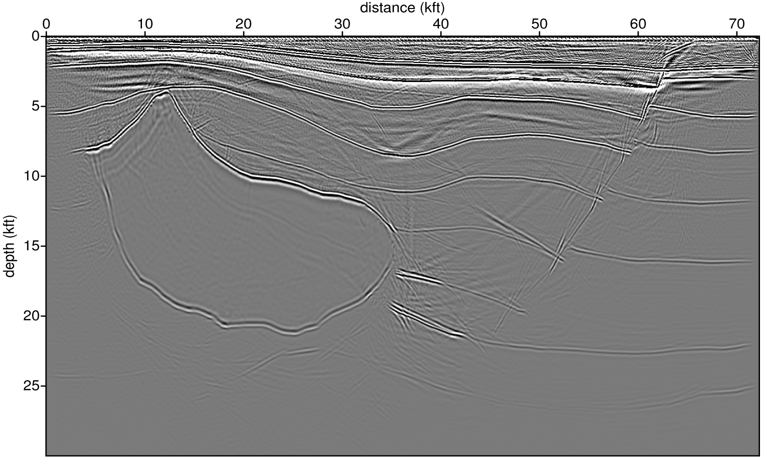


Hess VTI model

Delta parameter

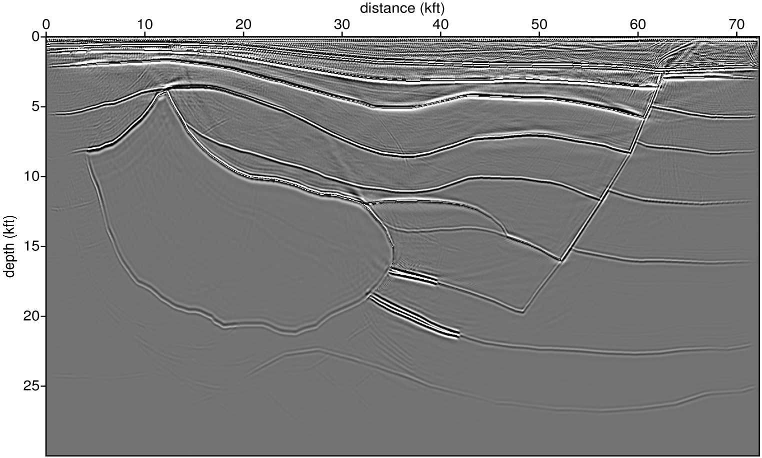


Isotropic RTM solved by REM (Δt=8 ms; F_{max}=35 Hz)



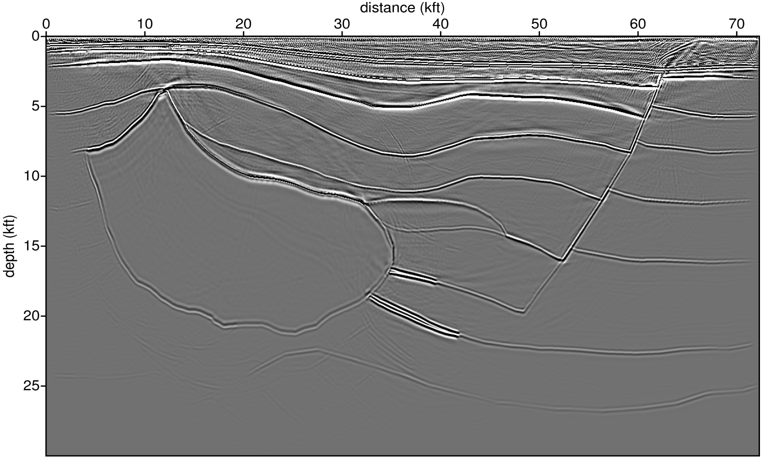
Hess VTI model

Anisotropic RTM solved by REM using using Du et al. (2008) equations



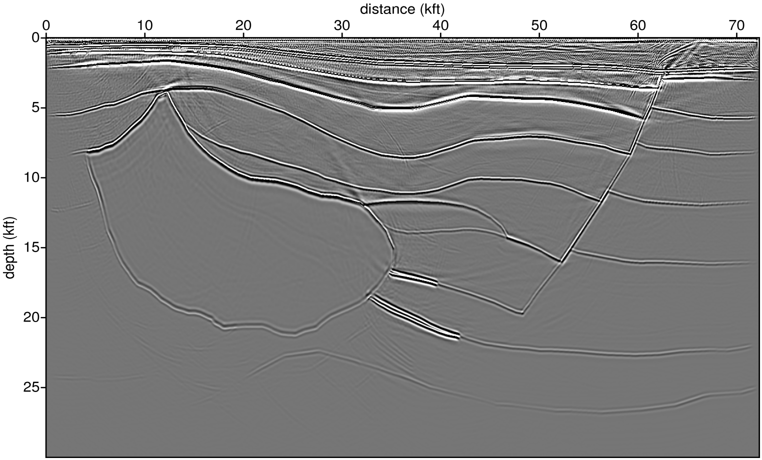
Hess VTI model

Anisotropic RTM solved by REM (pure P-wave)



Hess VTI model

Anisotropic RTM solved by REM using Etgen and Brandsberg-Dahl (2009)



Conclusions

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- The equations propose here can be effectively used in VTI media where $|\epsilon - \delta|$ is small. The SV-wave equation obtained is now well-posed and the triplication in the SV wavefront, as documented by Tsvankin (2001) is removed and allows a stable propagation.

Conclusions

- A new procedure to derive P- and SV-wave for VTI medium is proposed and analyzed.
- The equations propose here can be effectively used in VTI media where $|\epsilon - \delta|$ is small. The SV-wave equation obtained is now well-posed and the triplication in the SV wavefront, as documented by Tsvankin (2001) is removed and allows a stable propagation.
- The REM solution provides accurate and non-dispersive wave propagation and it was used to time-stepping the Du et al. (2008) system of equations and also the pure P scalar wave equation. The RTM REM for VTI media provides accurate images.

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