NTNU
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Kinematical parameters in 2D TTI media

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Outline

Theory

TTI traveltime parameters

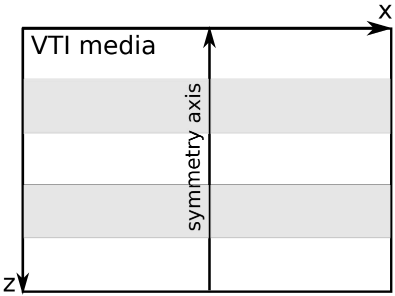
Weak-anisotropy approximation

Inversion

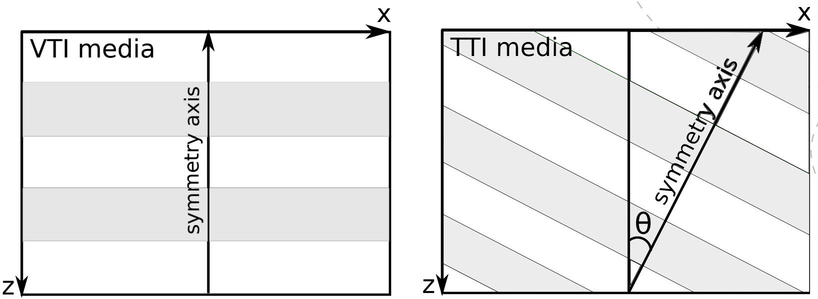
- Case with known tilt
- Surface seismic with check-shots
- Crosswell survey

Conclusions

Tilted TI media

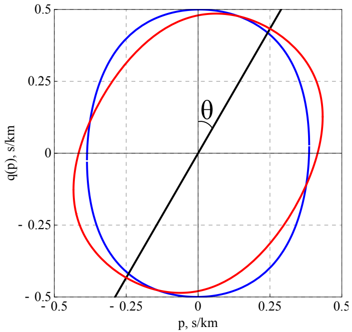


Tilted TI media



The symmetry axis of transversely-isotropic (TI) media can be not only vertical (VTI) or horizontal (HTI) but also tilted (TTI).

Slowness surface rotation



Slowness surfaces for VTI and TTI (tilt is 30°) media.

VTI slowness surface in acoustic approximation

$$q'^2_P - \frac{1}{v_0^2}\frac{1-(1+b_0)(1+a_0)p'^2v_0^2}{1-b_0(1+a_0)p'^2v_0^2} = 0$$

The rotation operator is defined as

$$\begin{pmatrix} p' \\ q' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} p \\ q \end{pmatrix}$$

where
v_0 is the symmetry-direction velocity,
$a_0 = 2\delta$, $b_0 = 2\eta = 2(\epsilon - \delta)/(1+2\delta)$
q and p are vertical and horizontal slowness, respectively

TTI traveltime parameters

Convenient parametrization:

$$\begin{aligned} R &= \cos^2\theta + (1+a_0)(1+b_0)\sin^2\theta \\ &= \cos^2\theta + \frac{v_{nmo}^2}{v_0^2}(1+2\eta)\sin^2\theta \end{aligned}$$

TTI traveltime parameters

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\end{aligned}
$$

$$
\begin{aligned}
Q &= b_0(1+a_0)\sin^2 2\theta \\
&= 2\eta\frac{v_{nmo}^2}{v_0^2}\sin^2 2\theta
\end{aligned}
$$

TTI traveltime parameters

$$t_0 = \frac{z}{v_0}\sqrt{\frac{2}{R+\sqrt{R^2-Q}}}$$

$$x_0 = \frac{z}{\sqrt{R^2-Q}}\left((R-1)\cot\theta - \frac{Q\cot 2\theta}{R+\sqrt{R^2-Q}}\right)$$

$$v_{nmo}^2 = v_0^2(1+a_0)\frac{\left(R+\sqrt{R^2-Q}\right)\left(2R-\sqrt{R^2-Q}\right)}{2(R^2-Q)^{3/2}}$$

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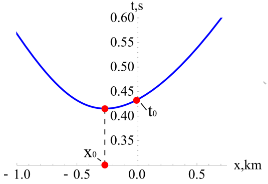

TTI traveltime parameters

$$S_2 = 1 + 8\frac{b_0(R^2 - Q)F_1 - QF_2}{\sqrt{R^2 - Q}\left(R + \sqrt{R^2 - Q}\right)\left(2R - \sqrt{R^2 - Q}\right)^2}$$

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where

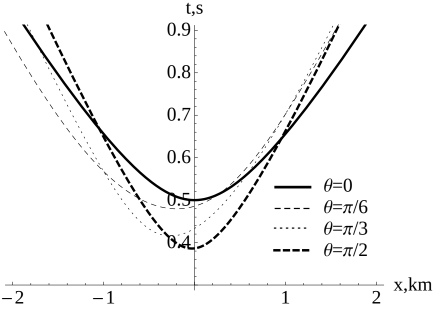
$$F_1 = R^2 + Q\frac{3R + \sqrt{R^2 - Q}}{R + \sqrt{R^2 - Q}}$$

$$F_2 = 2R^2 + Q\frac{3R + \sqrt{R^2 - Q}}{R + \sqrt{R^2 - Q}}$$

Special cases

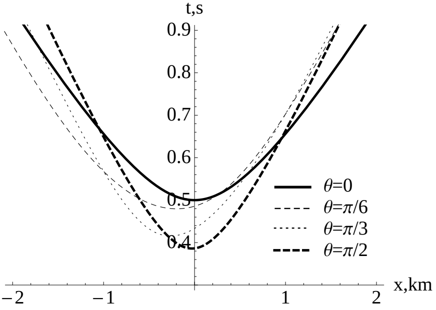
$VTI\ (\theta = 0^\circ)$	$HTI\ (\theta = 90^\circ)$	$ETI\ (b_0 = 0)$
$t_0 = \frac{2z}{v_0}$	$t_0 = \frac{2z}{v_0\sqrt{(1+a_0)(1+b_0)}}$	$t_0 = \frac{2z}{v_0\sqrt{1+a_0\sin^2\theta}}$
$x_0 = 0$	$x_0 = 0$	$x_0 = \frac{za_0\sin\theta\cos\theta}{1+a_0\sin^2\theta}$
$v_{nmo}^2 = v_0^2(1+a_0)$	$v_{nmo}^2 = \frac{v_0^2}{1+b_0}$	$v_{nmo}^2 = \frac{v_0^2(1+a_0)}{1+a_0\sin^2\theta}$
$S_2 = 1+4b_0$	$S_2 = 1+4b_0$	$S_2 = 1$

Reflection point sideslip

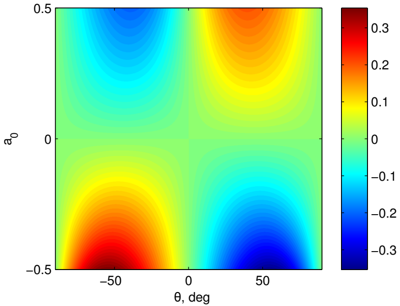


one-way traveltimes for different tilts

Reflection point sideslip



one-way traveltimes for different tilts



x_0 vs. a_0 and tilt θ ($b_0 = 0$)

Weak-anisotropy approximation

Assuming $|a_0| \ll 1$ and $|b_0| \ll 1$ we obtain:

$$t_0^2 \approx \frac{z^2}{v_0^2}\left(1 - a_0 \sin^2\theta - b_0 \sin^4\theta\right)$$

$$x_0 \approx z \cos\theta \sin\theta (a_0 + 2b_0 \sin^2\theta)$$

$$v_{nmo}^2 \approx v_0^2 (1 + a_0 \cos^2\theta + b_0 \sin^2\theta (6 - 7\sin^2\theta))$$

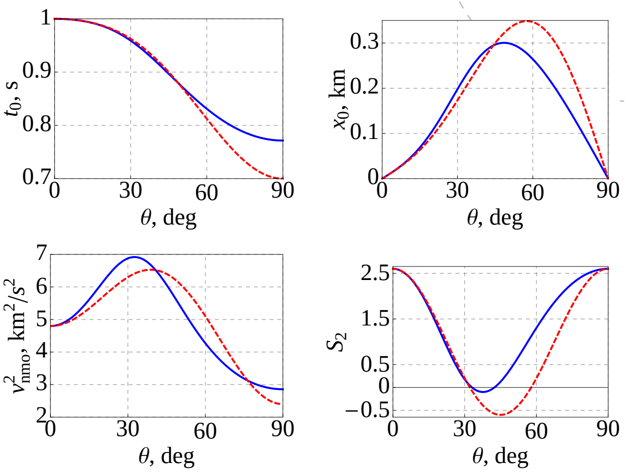
$$S_2 \approx 1 + 4b_0 \cos 4\theta$$

Weak-anisotropy approximation

$v_0 = 2\, km/s$

$a_0 = 0.2$

$b_0 = 0.4$



Exact

Approximated

Inversion of the traveltime parameters

We consider three cases:

- — surface seismic with known tilt θ
- — surface seismic with known symmetry-direction velocity v_0
- — cross-well survey with one-way traveltime parameters

Inversion of the traveltime parameters

We consider three cases:

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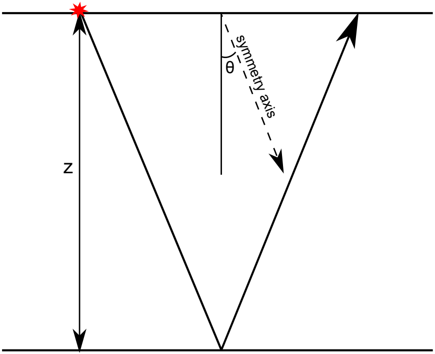
The model parameters are:
$z = 1km$, $v_0 = 2km/s$, $a_0 = 0.2$, $b_0 = 0.4$ (Model 1),
$z = 1km$, $v_0 = 2km/s$, $a_0 = 0.1$, $b_0 = 0.2$ (Model 2).

Case with known tilt



Known parameters:

— reflection depth z

— tilt θ

Estimated parameters:

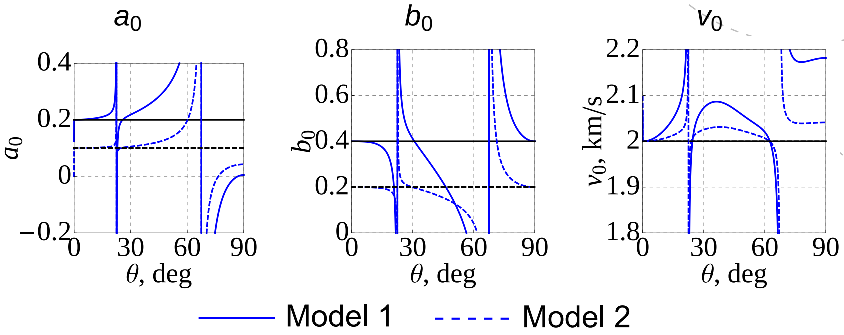
— t_0, v_{nmo}^2, S_2

Inverted parameters:

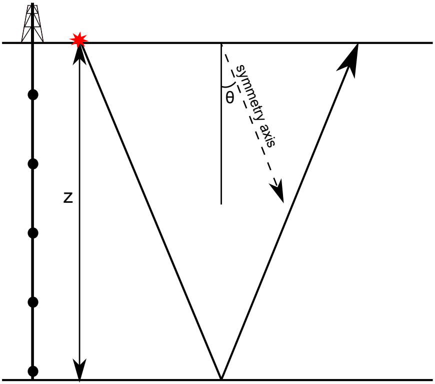
— anisotropic parameters a_0 and b_0

— symmetry-direction velocity v_0

Results of the inversion



Surface seismic with check-shots



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- reflection depth z
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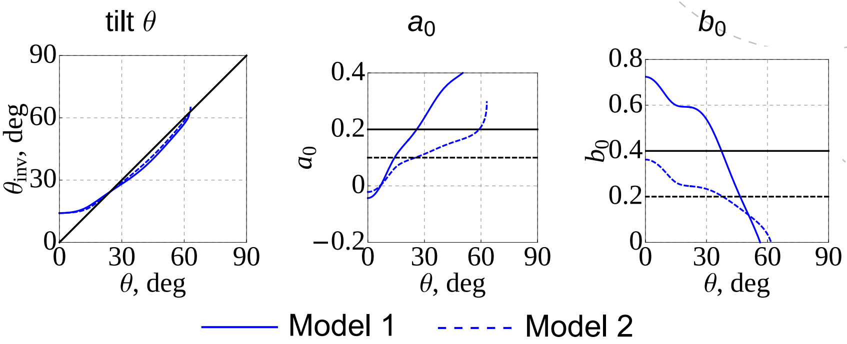
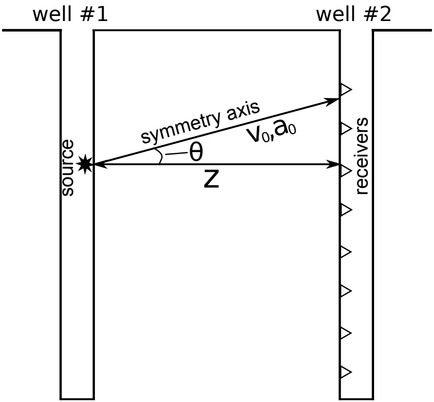


Crosswell survey



Known parameters:

— reflection depth z

Estimated parameters:

— t_0, v_{nmo}^2, S_2 and x_0

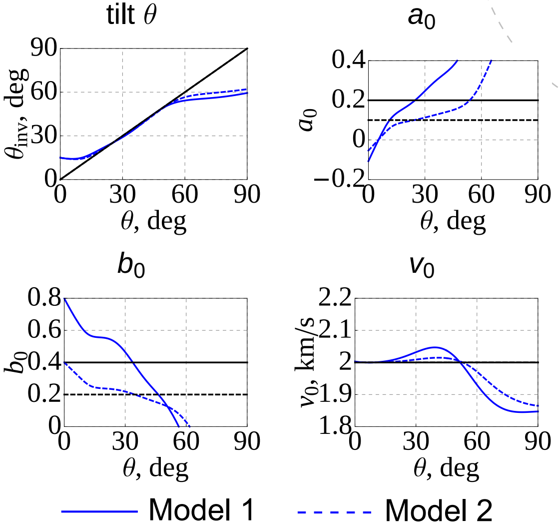
Inverted parameters:

— tilt θ

— anisotropic parameters a_0 and b_0

— symmetry-direction velocity v_0

Results of the inversion



Conclusions

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Conclusions

— Exact and WA-approximation traveltime parameters in TTI media are defined

— Inversion of traveltime parameters in WA approximation performed for three scenarios:

- tilt is known: the inversion is not defined for specific tilts $\cos 4\theta = 0$
- other scenarios: inversion for anisotropic parameters is unstable, but inversion for v_0 is good for $\theta \in [0, \pi/3]$

Acknowledgments

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Thank you!