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TIV Contrast Source Inversion of mCSEM data

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Overview

- Motivation
- Forward model
- Contrast Source Inversion (CSI)
- Results
- Conclusions

Motivation

- Many mCSEM surveys collected
- Not straightforward data interpretation
- Isotropic assumption not always sufficient

Motivation

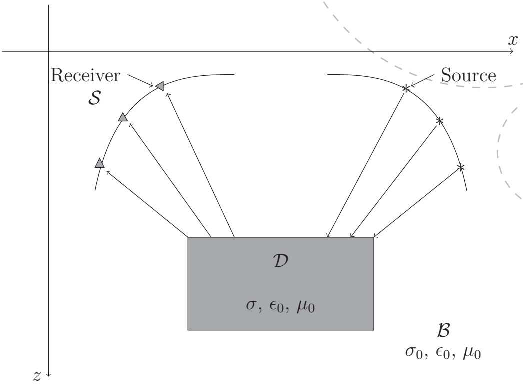
- Physical conditions may cause anisotropy
  - Grain orientation [Negi and Saraf, 1989]
  - Thin layering

- Consider TIV medium

$$\sigma = \begin{pmatrix} \sigma_h & 0 & 0 \\ 0 & \sigma_h & 0 \\ 0 & 0 & \sigma_v \end{pmatrix} = \sigma_v \begin{pmatrix} \Upsilon & 0 & 0 \\ 0 & \Upsilon & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Forward model

- Integral equations
- $\sigma_0, \epsilon_0, \mu_0$ denotes background parameters
- Domain $\mathscr{D}$ contains anomaly in σ



Forward model

– Field inside $\mathscr{D}$
[Abubakar and van den Berg, 2004]

$$\mathbf{e}_i(\mathbf{x}) = \mathbf{e}_i^{\mathrm{inc}}(\mathbf{x}) + \int_{\mathscr{D}} \mathbf{G}_{ij}^{E}(\mathbf{x},\mathbf{x}')\,\sigma_{0,v}(\mathbf{x}')\,\chi_{jj}(\mathbf{x}')\,\mathbf{e}_j(\mathbf{x}')\,\mathrm{d}\mathbf{x}'$$

– Scattered field at receivers

$$\mathbf{f}_i^{E}(\mathbf{x}) = \int_{\mathscr{D}} \mathbf{G}_{ij}^{E}(\mathbf{x},\mathbf{x}')\,\sigma_{0,v}(\mathbf{x}')\,\chi_{jj}(\mathbf{x}')\,\mathbf{e}_j(\mathbf{x}')\,\mathrm{d}\mathbf{x}'$$

$$\mathbf{f}_i^{H}(\mathbf{x}) = \int_{\mathscr{D}} \mathbf{G}_{ij}^{H}(\mathbf{x},\mathbf{x}')\,\sigma_{0,v}(\mathbf{x}')\,\chi_{jj}(\mathbf{x}')\,\mathbf{e}_j(\mathbf{x}')\,\mathrm{d}\mathbf{x}'$$

Forward model

– Contrast χ given by

$$\chi = \begin{pmatrix} \chi_h & 0 & 0 \\ 0 & \chi_h & 0 \\ 0 & 0 & \chi_v \end{pmatrix} = \begin{pmatrix} \frac{\sigma_h}{\sigma_{0,v}} - \Upsilon & 0 & 0 \\ 0 & \frac{\sigma_h}{\sigma_{0,v}} - \Upsilon & 0 \\ 0 & 0 & \frac{\sigma_v}{\sigma_{0,v}} - 1 \end{pmatrix}$$

Forward model

- Introduce contrast source $\mathbf{w} = \chi \mathbf{e}$
- Write equations in operator form

$$\begin{aligned} \mathbf{e} &= \mathbf{e}^{\text{inc}} + G^{E,\mathscr{D}}\mathbf{w} \\ \mathbf{f}^{E} &= G^{E,\mathscr{S}}\mathbf{w} \\ \mathbf{f}^{H} &= G^{H,\mathscr{S}}\mathbf{w} \end{aligned}$$

CSI

- Find contrast, χ, and set of contrast sources, $\mathscr{W} = \{\mathbf{w}^{j,k}\}_{j=1...N_s}^{k=1...N_f}$, that fit the equations given the data [Abubakar and van den Berg, 2004]

CSI

– Formally, minimize

$$
\begin{aligned}
& F_1\left(\mathscr{W},\chi\right) \\
= \quad & \alpha_1^E \sum_{k=1}^{N_\mathrm{f}} \sum_{j=1}^{N_\mathrm{s}} \left\| \Xi^{E,j,k} \left(\mathbf{f}^{E,j,k} - G^{E,\mathscr{S},k} \mathbf{w}^{j,k} \right) \right\|_{\mathscr{S}}^2 \\
+ \quad & \alpha_1^H \sum_{k=1}^{N_\mathrm{f}} \sum_{j=1}^{N_\mathrm{s}} \left\| \Xi^{H,j,k} \left(\mathbf{f}^{H,j,k} - G^{H,\mathscr{S},k} \mathbf{w}^{j,k} \right) \right\|_{\mathscr{S}}^2 \\
+ \quad & \alpha_2 \sum_{k=1}^{N_\mathrm{f}} \sum_{j=1}^{N_\mathrm{s}} \left\| \chi \mathbf{e}^{\mathrm{inc},j,k} - \mathbf{w}^{j,k} + \chi G^{E,\mathscr{D},k} \mathbf{w}^{j,k} \right\|_{\mathscr{D}}^2
\end{aligned}
$$

CSI

- In addition, introduce regularizer [Tikhonov and Arsenin, 1977]

$$
\begin{aligned}
F(\mathscr{W},\chi) &= F_1(\mathscr{W},\chi) + \lambda^2 F_2(\chi) \\
&= F_1(\mathscr{W},\chi) + \lambda^2 \|\Omega(\chi - \chi^{\mathrm{ref}})\|_{\mathscr{D}}^2
\end{aligned}
$$

- Minimized alternatingly w.r.t. $\mathscr{W}$ and χ

Example

– Single line data from Troll Field

– 37 receivers, inline electric and crossline magnetic field

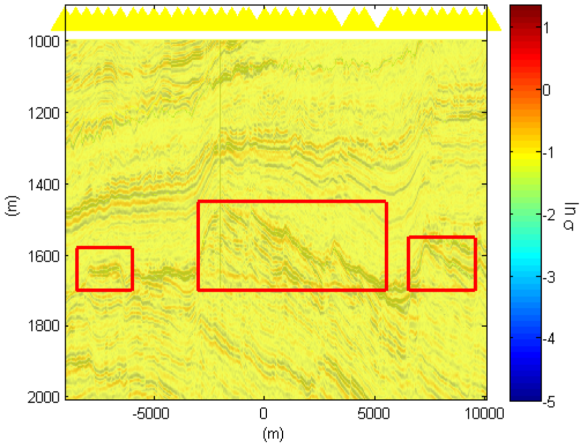
– 0.25Hz, 0.75Hz

– $\mathscr{D}: 22\mathrm{km} \times 2\mathrm{km} \times 1\mathrm{km}$

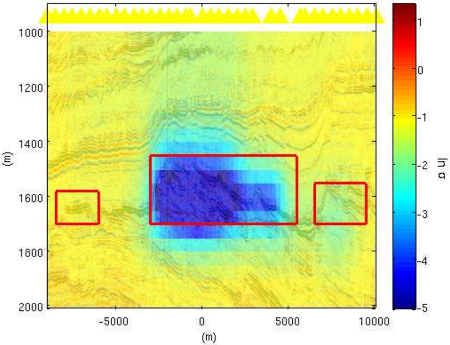
– $\Delta x = 250\mathrm{m}, \Delta y = 250\mathrm{m}, \Delta z = 50\mathrm{m}$

– Ω: approximation to Laplacian

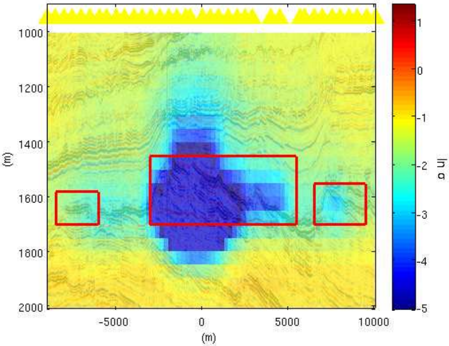
Troll


1000
1200
1400
1600
1800
2000
(m)
-5000
0
5000
10000
(m)
1
0
-1
-2
-3
-4
-5
ln σ

Troll

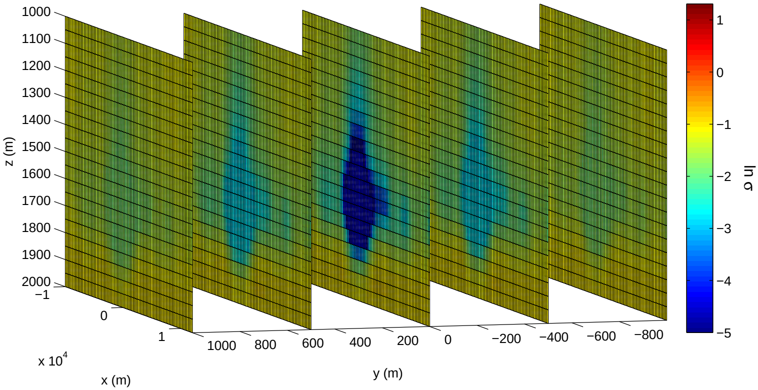


Isotropic



TIV, σ_V

Troll


1000
1100
1200
1300
1400
1500
1600
1700
1800
1900
2000
z (m)
−1
0
1
x 10^4
x (m)
1000
800
600
400
200
0
−200
−400
−600
−800
y (m)
1
0
−1
−2
−3
−4
−5
ln σ

Datafit

— Increased datafit with TIV due to additional degrees of freedom

	e	h
Isotropic	3.84%	2.34%
TIV	3.41%	2.26%

Conclusions

- TIV may be necessary in inversion
- Information concerning background is needed
- View in context with methods that discretize the entire survey area

References

- Abubakar, A., and P. M. van den Berg, 2004, Iterative forward and inverse algorithms based on domain integral equations for three-dimensional electric and magnetic objects: Journal of Computational Physics, **195**, 236–262.
- Negi, J. G., and P. D. Saraf, 1989, Anisotropy in geoelectromagnetism: Elsevier, New York.
- Tikhonov, A. N., and V. Y. Arsenin, 1977, Solution of ill-posed problems: W.H.Winston and Sons.