It is well known that if we are able to estimate acoustic impedance (AI) and a parameter related to shear-wave velocity from seismic data, our ability to discriminate between different lithologies and fluid phases will increase. Prestack inversion on individual CDP gathers and inverting directly for $V_p$, $V_s$, and density have been tested in several ways, but the estimated parameters are often poorly determined.

A more robust approach is to apply poststack inversion on partial stacks. For inversion of the near-offset stack, AI can be calculated directly from well logs. However, for the far-offset stack, we need to derive an equivalent of the acoustic impedance that can be used to calibrate the non-zero-offset seismic reflectivity. Connolly (1998, 1999) derived such an equivalent—elastic impedance (EI)—and demonstrated how, by using elastic impedance logs (which requires shear-wave logs), he was able to perform well calibration and inversion of far-offset data.

This article describes a new function—shear-wave elastic impedance (SEI)—for linking converted-wave stacks to wells using a linearization of the Zoeppritz equations. SEI is similar to EI but adapted to $P-S$ converted seismic data. It can be computed from acoustic log data ($P$- and $S$-wave velocities and density) and used for well calibration, wavelet estimation, and inversion of $P-S$ reflectivity data leading to improved interpretability of converted wave data. (Equation 4 in Appendix 1 is the key mathematical formula in this approach to SEI).

Example of SEI. To test the SEI technique, we used a 3-D four-component (4-C) ocean-bottom cable (OBC) survey acquired in 1997 that covers approximately 10 km$^2$ of the Statfjord Field (Rognø et al., 1999). Both $P$-wave and converted-wave data were 3-D prestack time-migrated. The hydrophone ($P$) and the vertical geophone ($Z$) component were summed to attenuate receiver side water-layer reverberations and both data sets ($PZ$ and $P-S$) were decimated. The summed $P$ and $Z$ data are hereafter denoted $PZ$ data.
The area used in the poststack inversion consists of 70 in-lines and 110 cross-lines with a common bin size of 25 × 25 m. The estimated angle range for the individual stacks is 5-25° for the \( \text{PZ} \) volume and 7-47° for the \( \text{P-S} \) volume.

Figure 1 shows several logs from well A. The two main reservoir units, the Brent Group and the Statfjord Formation, are mainly sandstone and shales. The Cook Formation is the only prolific unit within the Dunlin Group composed of sand interbedded with shales.

The EI and SEI logs were computed by integrating over the given angle ranges. In computing the SEI log, we estimated an average \( K = \frac{V_S}{V_P} \) value of 0.52 and an average incidence angle of about 27°. This explains the similarity between the shear-impedance (\( Z_s \)) and the SEI log in Figure 1. The EI and the SEI logs, scaled with an average value for the reservoir interval, are broadly similar.
logs at the wells. The computed elastic impedance and the shear elastic impedance volumes. In each case, data were matched to the common P-S wavelet. Except near Top Drake, Top Cook, and Top Nansen.

Figure 2 shows examples of PZ and P-S seismic stacks tied to elastic and shear elastic impedance logs in well A. The central part of the log interval (Top Dunlin [Drake] to Top Statfjord [Nansen]) has a good fit between synthetic seismograms and stacks. The discrepancy between the synthetic and real data near the top and bottom of the reservoir interval may be caused by the high deviation angle of the well (54°) and variation in data quality. The synthetic seismograms for EI and SEI are computed using 1-D convolution.

Wavelet analysis was done separately for the PZ and P-S volumes. In each case, data were matched to the computed elastic impedance and the shear elastic impedance logs at the wells. The PZ (green) and P-S (red) wavelets are close to zero phase (left side of Figure 3). The two wavelets have similar phase properties; however, there are significant frequency bandwidth differences. Their corresponding amplitude spectra are on the right of Figure 3. The passband of the PZ wavelet is centered at 20 Hz, and the P-S wavelet is centered at 10 Hz.

Figure 4 shows final processed PZ and P-S stacks for in-line 430. Note the significant difference in the frequency content. This makes time interpretation challenging because the seismic response of the reflecting interfaces within the PZ and P-S data appears quite different. The vertical resolution is, however, quite similar for the PZ and P-S volumes. There is a significant amplitude difference between the P-S and PZ synthetic seismograms (Figure 2) for the Cook interval. This amplitude difference is also apparent on the real seismic sections (Figure 4). Figure 6, a plot of EI versus SEI from well A, indicates that the sandy Cook interval will be easier to detect in the inverted P-S volume than the inverted PZ volume.

The corresponding inverted sections are shown (Figure 5) with overlays of EI and SEI logs from well A. The data were inverted using a sparse spike algorithm with constraints from the EI and SEI logs. In the SEI section, a large continuous impedance contrast separates the Cook Formation into low and high SEI layers. This impedance increase correlates with the porosity decrease through the Cook interval. A similar impedance boundary is less apparent in the inverted PZ volume. Another interesting feature is a continuous low impedance layer below Top Statfjord in the EI section. A similar feature is less apparent in the SEI section, which is expected due to a smaller SEI contrast across the Top Statfjord interface (Figures 2 and 3). This feature corresponds to the position of the gas-fluid contact in the upper part of the Statfjord Formation. Figure 7 compares seismically derived EI and SEI traces (blue) and their corresponding log impedance (red) at two blind wells. The seismically derived EI trace and the log-derived EI match very well especially within the reservoir below the Top Viking. A good match is also obtained between the seismically derived SEI trace and the log-derived SEI in the lowermost part of the reservoir interval.

**Estimation of Vp/Vs from inverted seismic data.** Two cubes are output from the poststack inversion of the OBC data: an elastic impedance cube and a shear elastic impedance cube. These were combined to compute an instantaneous Vp/Vs volume. Figure 8 shows a Vp/Vs cross-section of in-line 430 (position of line shown in Figure 9). Proper P-P and P-S event correlation is crucial because it has considerable effect on the outcome of this sample-by-sample based Vp/Vs computation. Prior to Vp/Vs computation, the SEI volume is transformed to P-P time using interval Vp/Vs ratios derived from the event correlation. In addition, to perform the computation, we need to choose a mean value for K (Vp/Vs = 0.5) and density (2.24 g/cm³).

To reduce the impact of minor mismatches in the event correlation, average interval Vp/Vs maps can be computed from average impedance maps. Figure 9 shows attribute maps extracted over the upper part of the Statfjord Formation. A band of low Vp/Vs trending nearly N-S appears in the eastern part of the map. This coincides with the position of the gas-fluid contact within the formation. A similar feature is seen as a low-impedance boundary on the EI map to the left but not on the SEI map (middle).

**Conclusions.** A new quantity derived from acoustic well log measurements, denoted shear-wave elastic impedance, is presented. Computing shear-wave elastic impedance in addition to acoustic and elastic impedance logs makes it possible to compare the relative strength between P-P and P-S events and to predict which interfaces might give strong P to S conversions. When Vp/Vs is close to 0.5 at incidence P-wave angles around 30°, the SEI log is approximately equal to the shear impedance log.

We have shown, with real data from the Statfjord Field, how SEI links converted-wave data to well logs. SEI can be used for P-S data as EI is used for P-P data. The technique is easy to implement in conventional interpretation packages where the SEI log can constrain the poststack inversion. The output from such an inversion could be directly compared with the shear-wave elastic impedance curve derived from acoustic well logs. Finally, elastic impedance and shear-wave elastic impedance can be combined to compute instantaneous Vp/Vs. The Vp/Vs estimate is based upon amplitude and may therefore serve as complementary (and more detailed) information for travel-time estimates of Vp/Vs.


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Appendix 1

Connolly (1998, 1999) derived the angle or offset equivalent of acoustic impedance, and used the term elastic impedance (EI). According to Connolly, EI is defined as:

\[
\text{EI}(\theta) = V_p \cdot 1 + \tan^2(\theta) V_s - 8 K^2 \sin^2(\theta) \frac{\Delta p}{\rho} 1 - 4 K^2 \sin^2(\theta) \frac{\Delta V_s}{V_s} \frac{\Delta V_s}{V_s}
\]

where EI(\theta) is the elastic impedance log at incidence P-wave angle \(\theta\); \(V_p\), \(V_s\), and \(\rho\) denote the P-wave and S-wave velocity and density, respectively; and \(K\) is the average \(V_s/V_p\).

The basic assumption about elastic impedance is that it should play the same role for angle-dependent P-wave reflectivity as AI does for zero-offset reflectivity. If that condition is fulfilled, one can apply poststack inversion schemes on partial stacks. For an incidence angle of zero, EI equals AI. However, note that the magnitude and units of EI vary with angle. VerWest and Masters (2000) presented a dimensionless version of this equation. EI logs are now being used as a quick way of assessing AVO characteristics expected from a given well, and as a tool for calibration and inversion of partial stack volumes.

An approximate equation (assuming weak contrasts and small angles) for P to S reflectivity, \(R_{ps}(\theta_p)\), as a function of incidence P-wave angle at a plane interface is:

\[
R_{ps}(\theta_p) = \frac{1}{2} \left[ (1 + 2 K) \frac{\Delta p}{\rho} + 4 K \frac{\Delta V_s}{V_s} \right] \sin(\theta_p) + K \left[ (1 + 2 K) \frac{\Delta p}{\rho} + 2 \frac{\Delta V_s}{V_s} \right] \Delta p \rho \sin^3(\theta_p)
\]
where

\[ \Delta \rho = \rho \Delta \rho \quad \text{and} \quad \Delta V_s = V_s \Delta V_s \]

denote the fractional changes in density and S-wave velocity across an interface and the bar denotes the average.

To relate SEI to P-S reflectivity in the same way that AI relates to P-P reflectivity, the following equation must be fulfilled:

\[ R_{ps}(-\theta_p) = -R_{ps}(\theta_p) = \frac{SEI_2(\theta_p) - SEI_1(\theta_p)}{SEI_2(\theta_p) + SEI_1(\theta_p)} \]

(3)

Subscripts 1 and 2 refer to the layer above and below the interface, respectively. To derive SEI with a similar depth trend as acoustic, elastic, and shear impedance, we choose to use the negative offsets of CCP (common conversion point) gathers. A one-term version of SEI (using only the first term in equation 2 in the derivation) is given by Landrø et al. (1999). A higher-order approximation is obtained by combining equations 2 and 3

\[ SEI(\theta_p) = \frac{V_s}{\rho} m(K, \theta_p) n(K, \theta_p) \]

(4)

where

\[ m(K, \theta_p) = 4K \sin(\theta_p) \left[ 1 - \frac{1}{2}(1 + 2K) \sin^2(\theta_p) \right] \]

\[ n(K, \theta_p) = (1 + 2K) \sin(\theta_p) \left[ 1 - \frac{K(1 + 3K)}{(1 + 2K)^2} \sin^2(\theta_p) \right] \]

K is assumed constant in the derivation of equation 4. For an incidence angle of zero, SEI(\theta_p=0) equals 1 (no contrasts), corresponding to no P-to-S conversion (as expected). Figure A1 compares P-S reflectivity using the one-term SEI equation, the two-term SEI equation, and Zoeppritz equation for a two-layer model. For incidence P-wave angles above 25°, the one-term SEI equation is probably not sufficiently accurate. One should bear in mind that most converted waves are generated at higher incidence angles, hence we should use the two-term SEI equation (4).

A simpler P-S reflectivity function using only the shear impedance contrast can be written

\[ R_{ps}(\theta_p) = -\frac{\Delta Z_s}{Z_s} S(\theta_p) \]

(5)

where

\[ \frac{\Delta Z_s}{Z_s} \]

denotes the shear impedance contrast and

\[ S(\theta_p) = \sin \theta_p \cos^2 \theta_p \]

is an angle dependent scalar.

It is assumed in this derivation that the density contrast is small compared with the shear impedance contrast, and that the average \( V_s/V_p \) ratio equals 0.5. Figure A2 illustrates the P-S reflectivity error between equations 5 and 3 at incidence angles of 10°, 30°, and 40°. The error curves are computed by varying only the \( V_p \). Given the model in Figure A1 (\( V_s/V_p = 0.6 \)), the error is approximately -15% for all three angles. For an incidence angle of 30° and K equal to 0.5, SEI is approximately equal to the shear impedance. For other incidence angles and K values different from 0.5, the error in P-S reflectivity increases.