# Reverse-time demigration using the extended imaging condition

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## SUMMARY

The two-way process of seismic migration and demigration or remodeling has many useful applications in seismic data processing. We present a method to re-obtain the seismic reflection data after migration, by inverting the common image point gathers produced by reverse-time migration with an extend imaging condition. This allows to convert the results of seismic data processing in the stacked image domain back to the prestack reflection data domain. To be able to reconstruct the data with high fidelity, we set up demigration as a least squares inverse problem, and solve it iteratively using a steepest descent method. Because we use an extended imaging condition, the method is not dependent on an accurate estimate of the migration velocity field, and is able to accurately reconstruct both primaries and multiples. Numerical results show the feasibility of the method, and highlight some of its applications on 2D synthetic data sets.

## INTRODUCTION

The motivation behind this work was to obtain a method to reconstruct seismic reflection data from common image point gathers (CIGs) constructed with reverse-time migration (RTM). The method should work without the need for an accurate velocity model, and the reconstructed data should have an acceptably small error in amplitude and phase. This would ultimately allow us to process data in the migrated domain, which can be an advantage in many situations, such as velocity analysis, illumination studies, data interpolation and multiple attenuation.

Demigration methods have a long history in seismic data processing. Loewenthal et al. (1976) introduced the concept of an exploding reflector model, showing how to obtain zero-offset seismic data from a migrated stack using a background velocity model and wave theoretical methods. The Kirchhoff integral and the high frequency approximation have also been used for reconstruction of seismic data form migrated images (Jaramillo and Bleistein, 1999; Santos et al., 2000; Miranda, 2006). More recently, RTM has been used to recreate data from seismic images with the purpose of velocity analysis (Chauris and Benjemaa, 2010), and multiple attenuation (Zhang and Duan, 2012).

In their work, Chauris and Benjemaa (2010) use the extended imaging condition (Sava and Vasconcelos, 2011) in a migration/demigration scheme. The advantage of the extended imaging condition over the classical cross-correlation imaging condition (Claerbout, 1971), is that it preserves the phase and angle dependent amplitude information of the data in the migrated image, even in the case of migration with an inaccurate velocity model. Here we explore this fact and use extended images to set up demigration as an inverse problem. We try to reconstruct the prestack seismic reflection data from the migrated image by minimizing a least-squares function. And we solve the problem iteratively using a steepest descent method.

Results from 2D synthetic numerical examples show that phase information can be recovered after only one iteration, whereas the amplitude information can require many steepest descent iterations.

## THEORY

The main purpose of the method is to be able to reconstruct seismic data from CIGs constructed using RTM with an extend imaging condition (Sava and Vasconcelos, 2011). In the extended imaging condition, instead to Claerbout (1971) classical cross-correlation of the source and receiver wavefields at the imaging point, CIGs are constructed by cross-correlating the source and receiver wavefields at symmetric lags around the imaging point. These cross-correlation lags can be either spatial (Rickett and Sava, 2002) or temporal lags (Sava and Fomel, 2006). The important point is that, contrary to the classical imaging condition, the extended imaging condition preserves the kinematic information of the data in the image, even in the case of migration with an inaccurate velocity model. This characteristic has been extensively exploited in automatic velocity analysis methods (Shen et al., 2003; Sava and Biondi, 2004; Mulder, 2008; Chauris and Benjemaa, 2010; Weibull and Arntsen, 2011). We now show how we can use the extended imaging condition to set up a demigration method able to recover both amplitude and phase of the seismic reflection data. We develop the method using a time domain implementation of RTM with a space-lag cross-correlation imaging condition (Rickett and Sava, 2002)

$$R^0(\mathbf{x},\mathbf{h}) = \int ds \int dt\; W_s(\mathbf{x}-\mathbf{h},t,s) \times \int d\mathbf{x}' \int dt'\; G(\mathbf{x}+\mathbf{h},t;\mathbf{x}',t')P^0(\mathbf{x}',t',s), \quad (1)$$

where $R^0$ are CIGs (extended image), $\mathbf{x} = (x,y,z)$ are the spatial coordinates, $\mathbf{h} = (h_x,h_y,h_z)$ are spatial lags, $t$ is the time, $s$ is the source index, $W_s$ are the source wavefields, $G$ is the acoustic Green's function, $P^0(\mathbf{x}',t',s)$ are common shot gathers.

The source wavefields are given by

$$W_s(\mathbf{x},t,s) = \int d\mathbf{x}' \int dt'\; G(\mathbf{x},t;\mathbf{x}',t')S(\mathbf{x}',t',s), \quad (2)$$

where $S$ are source functions.

Assume now that we have the CIGs ($R^0$), and we would like to obtain the data ($P^0$), that is, we are interested in the inverse procedure of equation 1. One approach is to apply the adjoint of migration, which, for the extend imaging condition, can be



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written as

$$P(\mathbf{x}',t',s) = \int d\mathbf{x}' \int dt'\ G(\mathbf{x},t;\mathbf{x}',t') \times \int d\mathbf{h} \frac{\partial^2 R^0}{\partial z^2}(\mathbf{x}'-\mathbf{h},\mathbf{h}) W_s(\mathbf{x}'-2\mathbf{h},t',s). \quad (3)$$

A similar equation has been used by Chauris and Benjemaa (2010) for velocity analysis. One problem with this modeling equation is that, even if it properly recreates the kinematics, it gives the wrong amplitudes for the data. Another approach, and the one that is proposed by this paper, is to cast the problem as a least squares inversion of the following objective function

$$J = \frac{1}{2} \int d\mathbf{x} \int d\mathbf{h} \left[ \frac{\partial R^0}{\partial z}(\mathbf{x},\mathbf{h}) - \frac{\partial R}{\partial z}(\mathbf{x},\mathbf{h}) \right]^2. \quad (4)$$

Here $R^0$ are the CIGs to be demigrated, and $R$ are forward mapped CIGs. The forward mapped CIGs are computed according to

$$R(\mathbf{x},\mathbf{h}) = \int ds \int dt\ W_s(\mathbf{x}-\mathbf{h},t,s) \times \int d\mathbf{x}' \int dt'\ G(\mathbf{x}+\mathbf{h},t;\mathbf{x}',t') P(\mathbf{x}',t',s), \quad (5)$$

where, $P(\mathbf{x}',t',s)$ are the unknown shot gathers to be estimated. Note that $R$ is migrated using the same source wavefields $W_s$ as $R^0$. The vertical spatial derivatives in equation 4 are used to remove well-known artifacts from RTM images (Guitton et al., 2007).

By minimizing this objective function, we seek to find the data that, when migrated, will approximate the image migrated with the original data in a least squares sense. In principle, because of the linear relationship between the data and the receiver wavefields, the problem is linear and its solution can be sought explicitly. However, here we choose to solve the proposed least squares problem using a steepest descent method (Nocedal and Wright, 2000). This means that the demigrated shot gathers are updated iteratively according to

$$P_{i+1}(\mathbf{x},t,s) = P_i(\mathbf{x},t,s) - \alpha_i \frac{\partial J}{\partial P_i}(\mathbf{x},t,s), \quad (6)$$

where $i \in (1,...,N)$ is the iteration index, $\alpha_i$ is a positive step length, and the gradient $\frac{\partial J}{\partial P_i}$ is given by

$$\frac{\partial J}{\partial P_i}(\mathbf{x},t,s) = \int d\mathbf{x}' \int dt'\ G(\mathbf{x},t;\mathbf{x}',t') \times \int d\mathbf{h} \frac{\partial^2 \Delta R_i}{\partial z^2}(\mathbf{x}'-\mathbf{h},\mathbf{h}) W_s(\mathbf{x}'-2\mathbf{h},t',s), \quad (7)$$

with the image residual $\Delta R_i$ being given by

$$\Delta R_i(\mathbf{x},\mathbf{h}) = R^0(\mathbf{x},\mathbf{h}) - \int ds \int dt\ W_s(\mathbf{x}-\mathbf{h},t,s) \times \int d\mathbf{x}' \int dt'\ G(\mathbf{x}+\mathbf{h},t;\mathbf{x}',t') P_i(\mathbf{x}',t',s). \quad (8)$$

## NUMERICAL RESULTS

We illustrate the method with 2D seismic examples. The examples are based on the Marmousi model (Versteeg, 1993). The Marmousi acoustic model, shown in Figure 1, is used to generate seismic data using a finite difference modeling code (Virieux, 1986). We use a monopole point source and Ricker wavelet with dominant frequency of 20 Hz. The source spacing, and the receiver spacing are both 25 m. The shot gathers have a minimum offset of 0 km and a maximum offset of 5 km.

### Example 1

In the first example, we show the ability of the method to reconstruct the seismic prestack data from stacked migrated CIGs. In this procedure, the CIGs that are input for demigration, are the CIGs that are output from RTM with the original data, without any modification. The data that is output from demigration is then compared to the original data after 1, and after many iterations. The migration velocity model is shown in Figure 2. The migrated stacked image (zero lag) and a collection of CIGs are shown in Figure 3. The demigration is carried out without knowledge of the original data, except for its geometry. In other words, the initial shot gathers are a collection of zeroed traces.

Figure 4 shows a comparison of a particular shot gather at source position 7.83 km of the original data with the result of demigration of the CIGs after one iteration, and after 5 iterations of demigration. In this comparison, the reconstructed shot gathers were scaled by an optimal scalar constant. This constant was found by minimizing the least squares difference of the amplitudes between the reconstructed shot gathers and the original shot gathers.

Figure 5 shows a comparison of traces from the original data, and from the demigrated data after 1 iteration, and after 5 iterations of demigration. The results show that the kinematics of the data are reconstructed already after one iteration. And after 5 iterations, the data amplitudes are getting closer to the ones in the original data, as can be seen in the comparison of the time traces (Figures 5a-c), as well as in the comparison of the amplitude spectra (Figure 5d).

### Example 2

In the second example, we explore the application of demigration to interpolation of data. In this application, the Marmousi data of the previous example are decimated by only taking every 8th receiver. The resulting receiver interval of 200 m introduce severe dip aliasing in the seismic data recording. Migration using the decimated data results in CIGs such as the one shown in Figure 6a. The energy in the CIG that is outside the red dotted lines represents the migrated aliased events. This can clearly be concluded after comparing this CIG with the ones in Figure 3b, which were migrated with the original non-decimated geometry. Trying to reconstruct the data with the aliased energy in the CIGs will, in principle, reconstruct the aliased events in the data. However, we want to reconstruct the data without aliasing. To achieve that we mute the aliased energy from the CIGs (Figure 6b).

After mute, we demigrate the muted CIGs, acquiring the data




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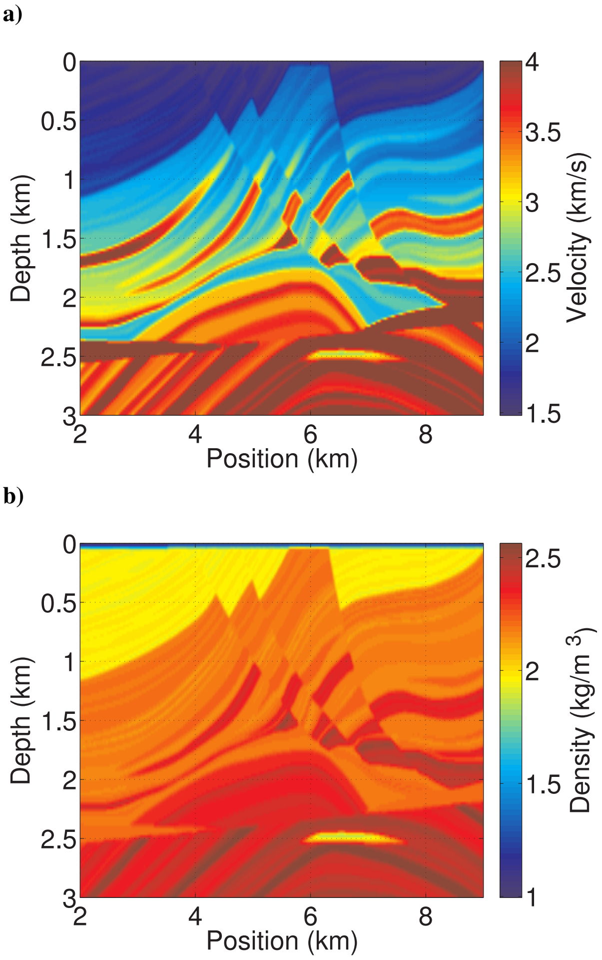


Figure 1: Marmousi acoustic model. a) Velocity model. b) Density model.

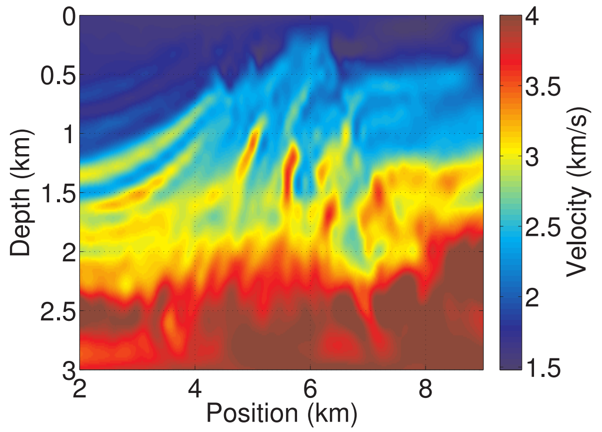


Figure 2: Migration velocity model for the Marmousi data set.

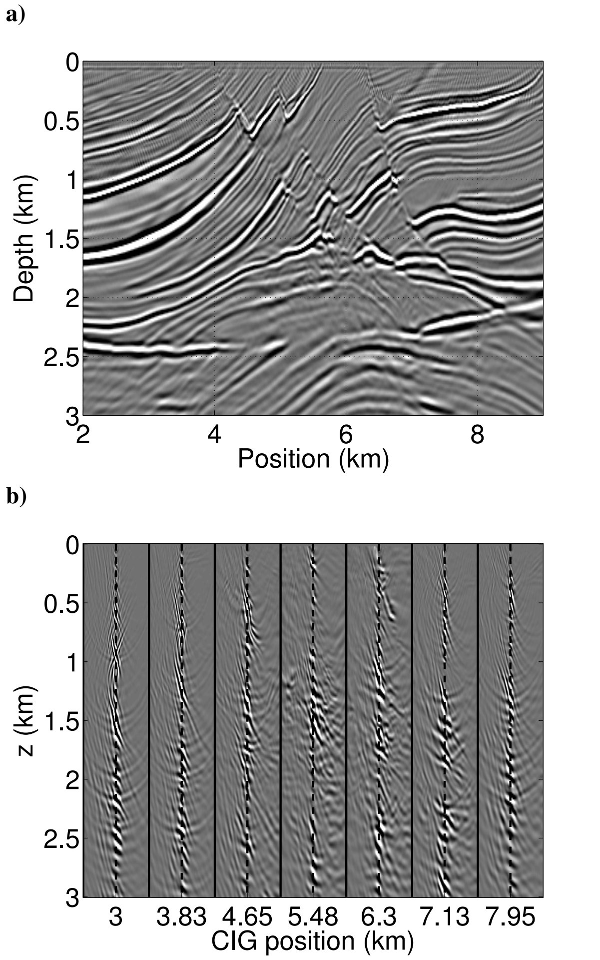


Figure 3: Marmousi migrated a) zero-lag image, and b) CIGs at several selected spatial positions.

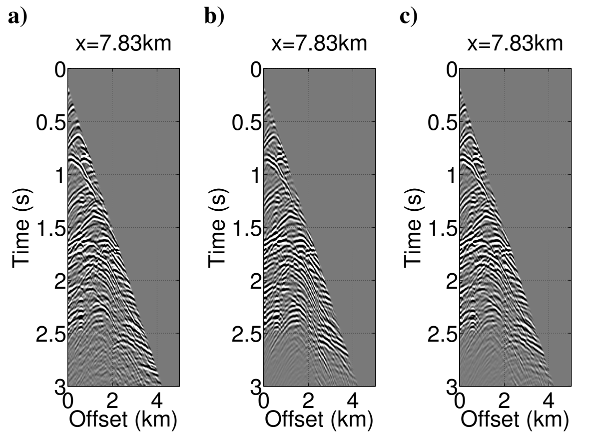


Figure 4: Marmousi shot gathers a) original, b) reconstructed after 1 iteration of demigration, c) reconstructed after 5 iterations of demigration.



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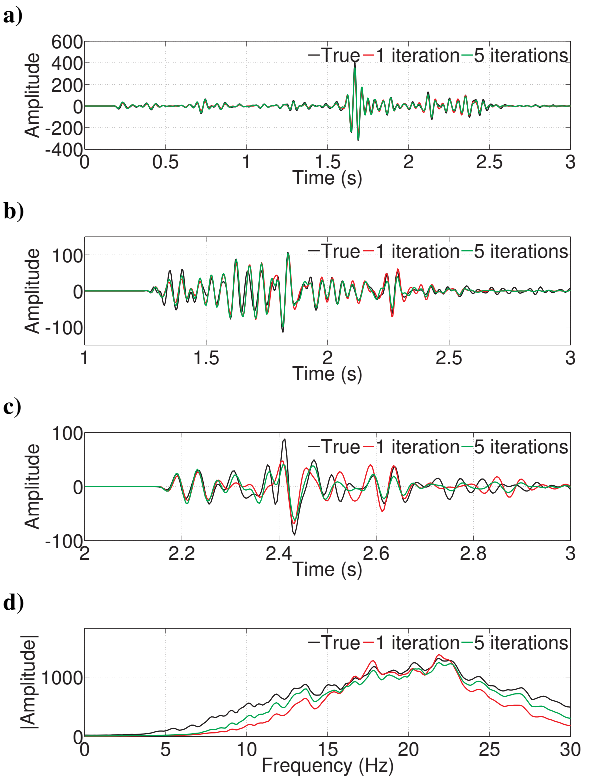


Figure 5: Comparison of traces of the Marmousi shot gather at source position 7.83 km a) at zero offset, b) at 1.65 km offset, c) at 3 km offset; d) Comparison of amplitude spectra averaged over all traces of the shot gather.

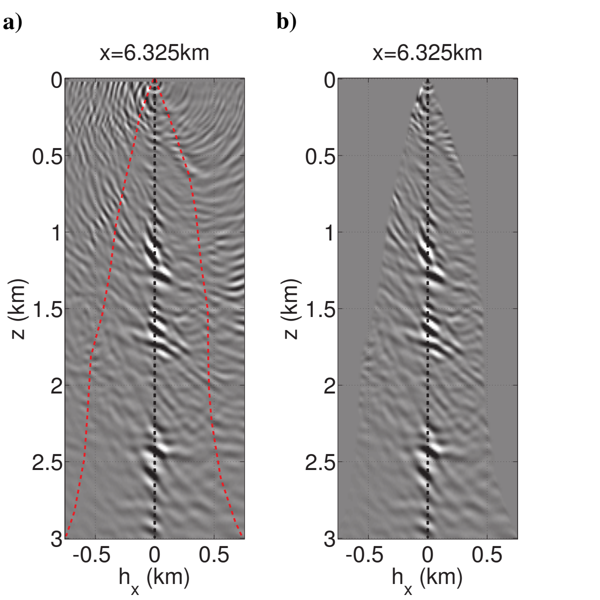


Figure 6: CIG at position 6.325 km a) before mute, b) after mute to remove aliased events. The dotted red lines in a) mark the position of the picked mute.

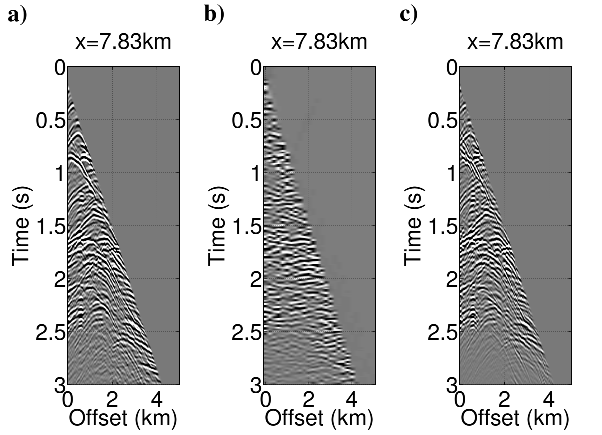


Figure 7: Shot gathers a) original, b) decimated, c) reconstructed shot gather after 5 iterations of demigration.

at the original receiver geometry. A comparison between the original shot gather, the decimated shot gather, and the reconstructed shot gather is shown in Figure 7.

Results show that demigration successfully reconstructs the kinematics of the original shot gather. Due to the smaller fold of the stack of the image constructed with the decimated data, the amplitudes of the reconstructed data will tend to converge to different values from those of the original data (about 8 times smaller in this case). Also, the kinematic reconstruction of the aliased events are biased by the migration velocity model, possibly causing the reconstructed aliased events to have different moveouts than in the original data. Despite these limitations, this type of reconstruction can find useful application in interpolating irregularly sampled aliased data, where most classical methods fail (Zwartjes and Sacchi, 2007).

## CONCLUSION

We presented a method to reconstruct seismic reflection data from stacked CIGs constructed through RTM with an extended imaging condition. The method is based on least squares inversion, and is solved iteratively using a steepest descent approach. The numerical examples show that the extended imaging condition allows kinematic reconstruction of the prestack seismic reflection data after only one iteration, although many iterations are required to recover the correct amplitudes. The presented method has many interesting applications, such as image-based demultiple and data interpolation. And because the method is based on RTM, it can be applied to process data acquired over complex geological media.

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**EDITED REFERENCES**

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