Dynamic Time Warping
–
an improved method for time lapse and tomography time shift estimation?

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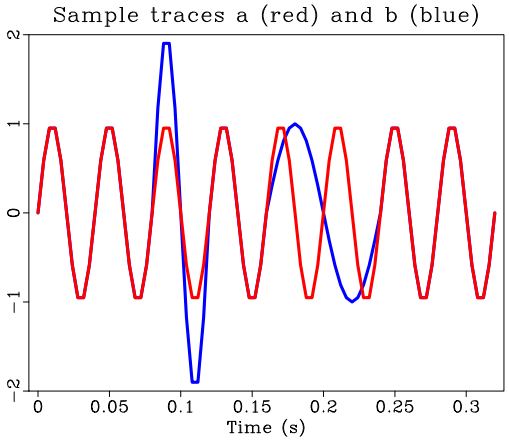
Houston
September 24

Time shifts — What are they?



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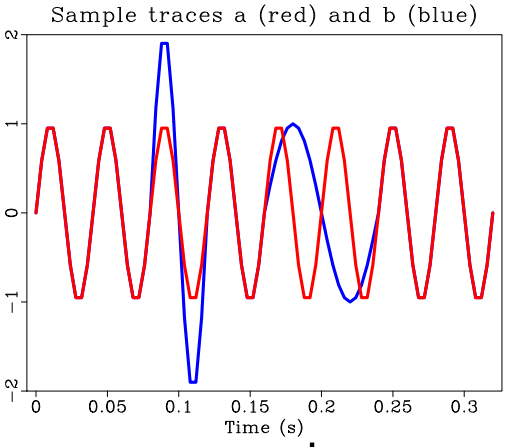

$b(t) - a(t)$

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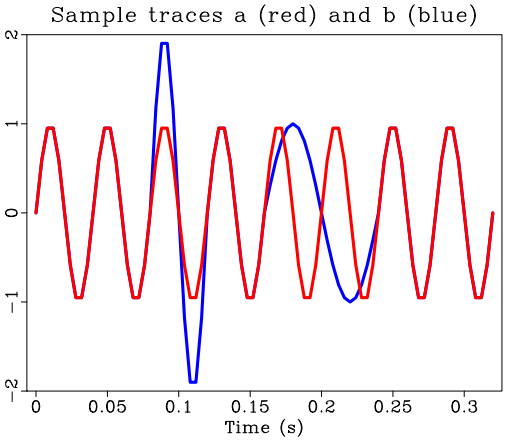

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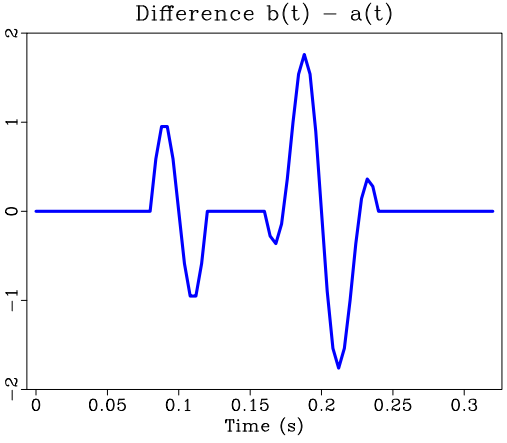

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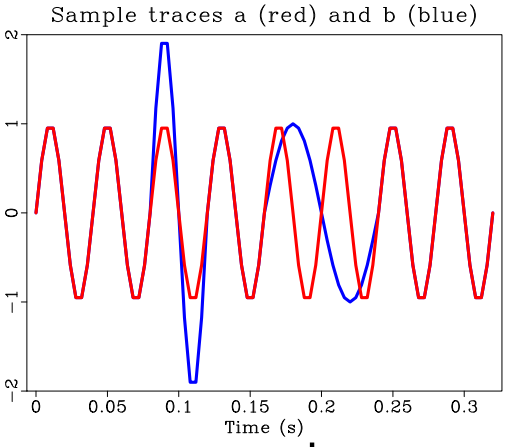

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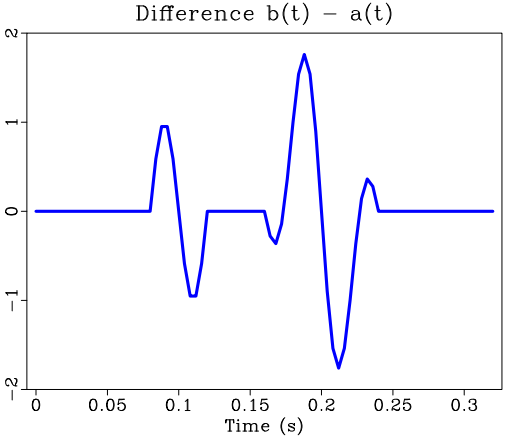

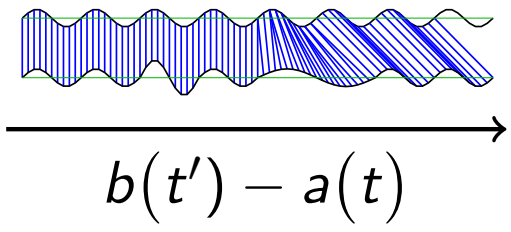

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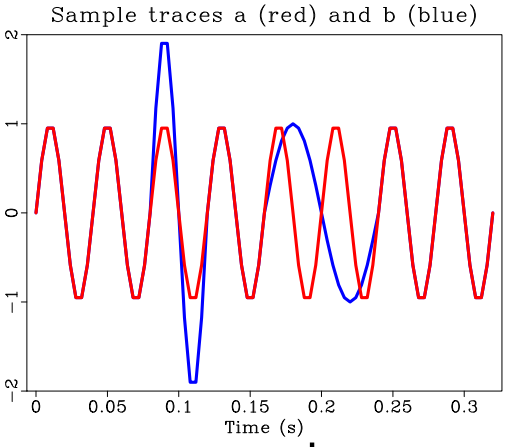

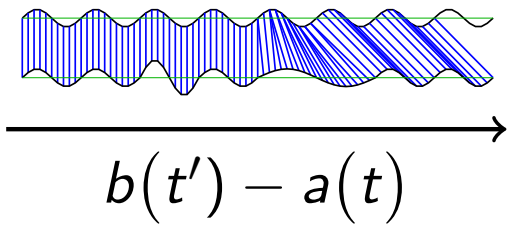

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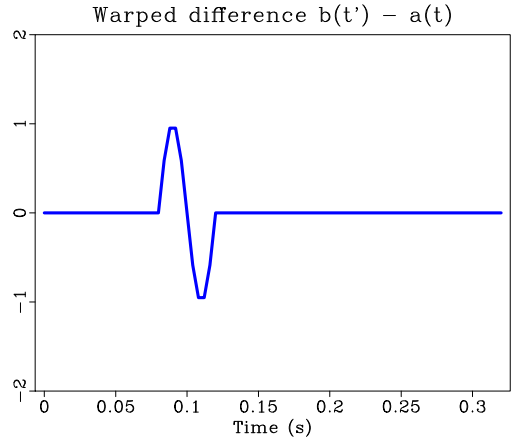

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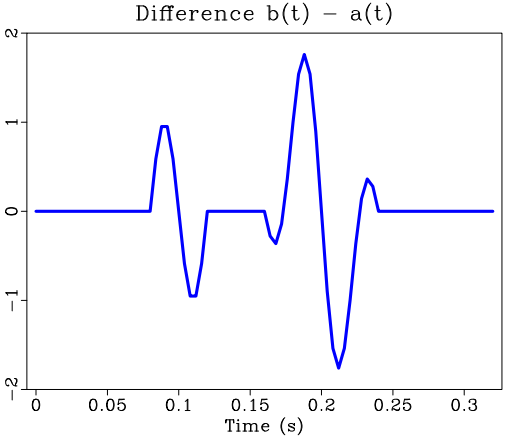

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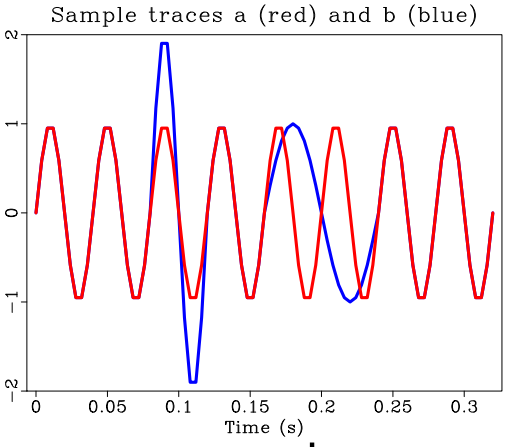

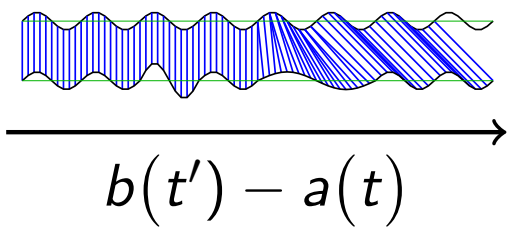

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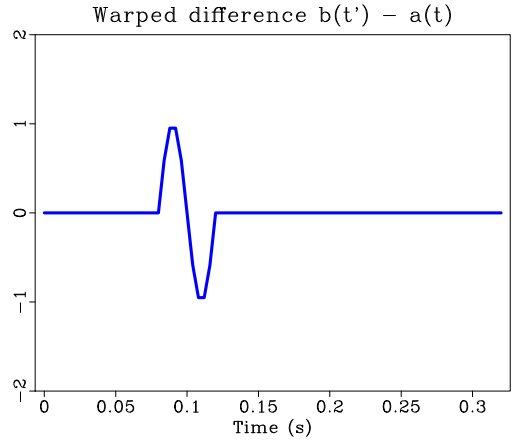

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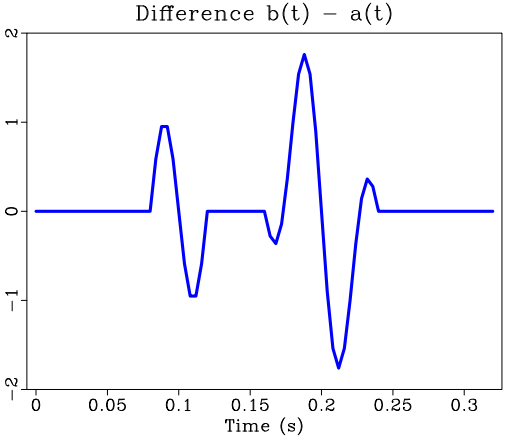

Time shifts are exactly $t' - t$

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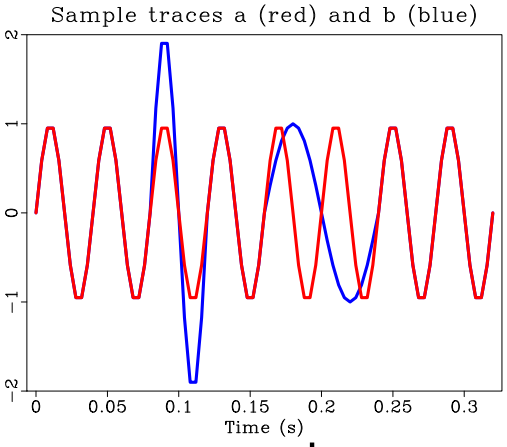

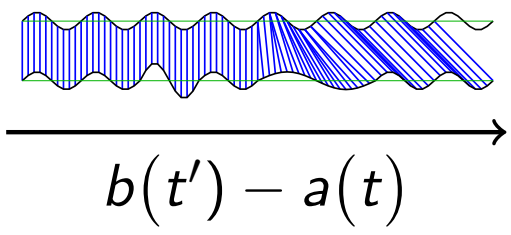

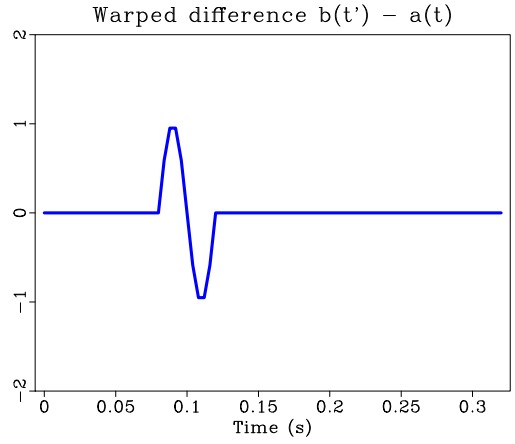

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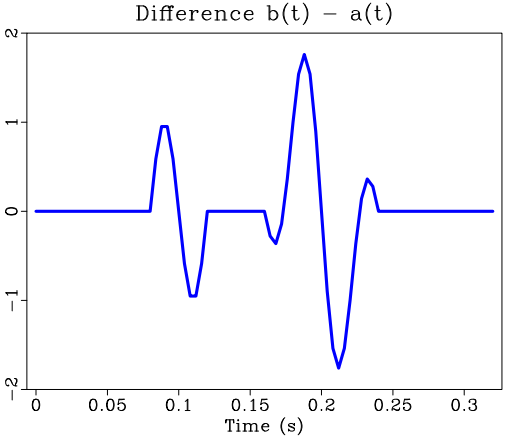

Difference: $b(t) - a(t)$

Time shifts are exactly $t' - t$

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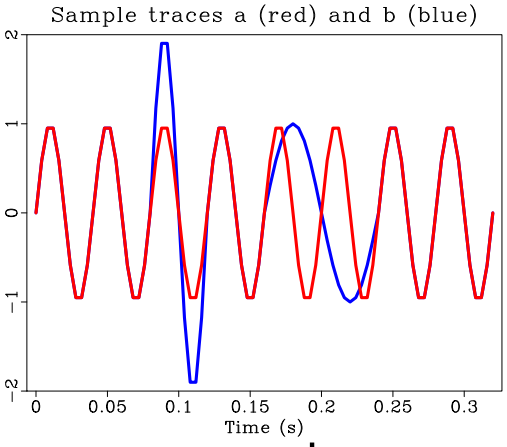

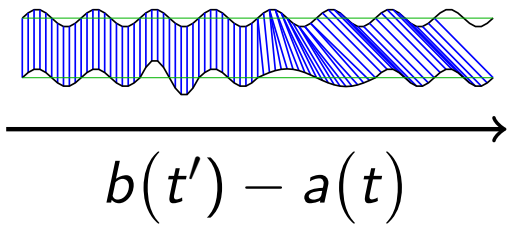

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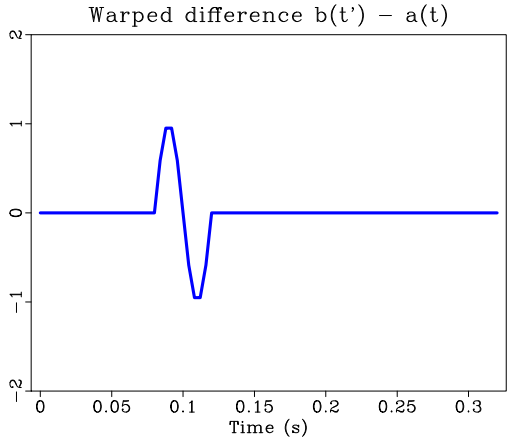

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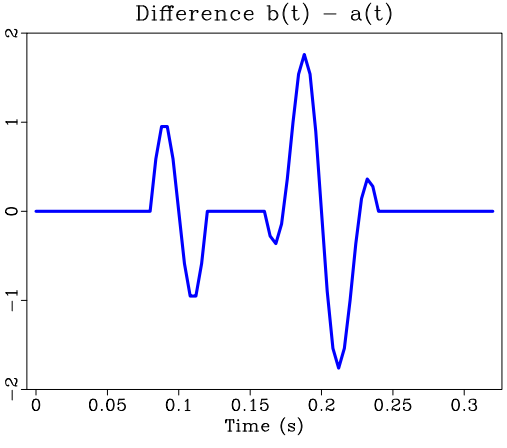

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Time shifts: $t'\big(a(t)\big) - t$

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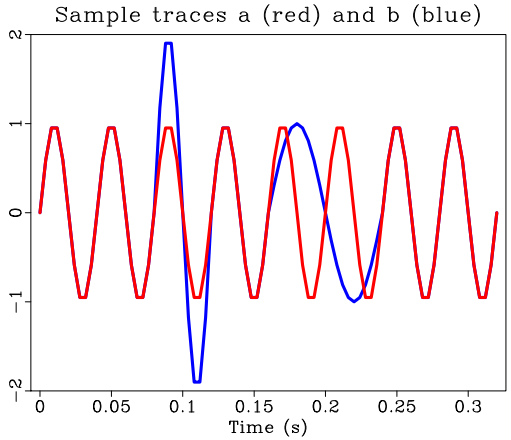
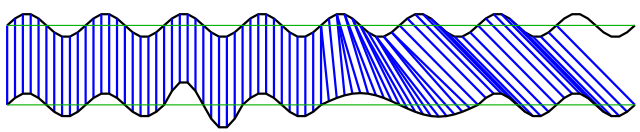


Correct time shifts.



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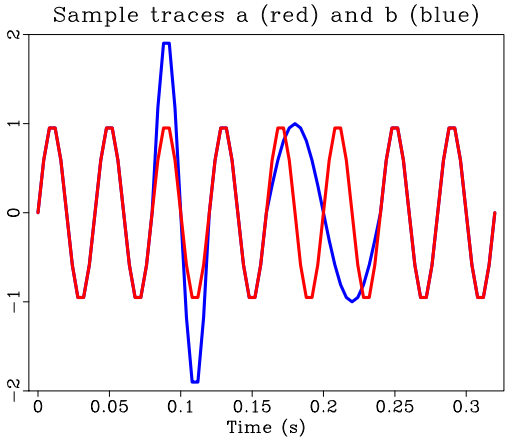
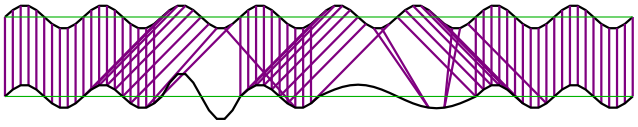


"Naïve" guess.



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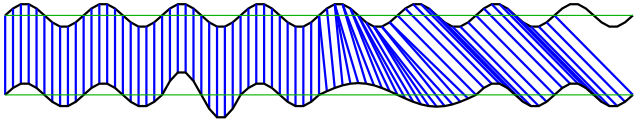

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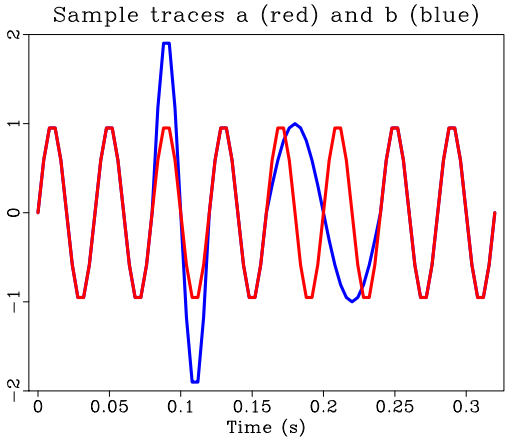

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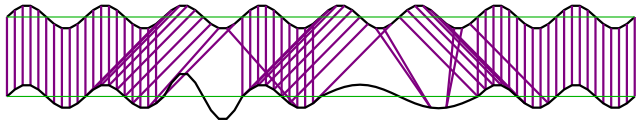

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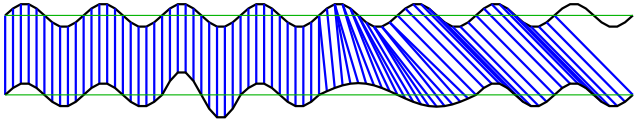
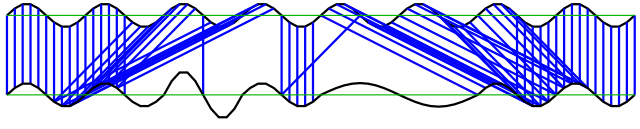
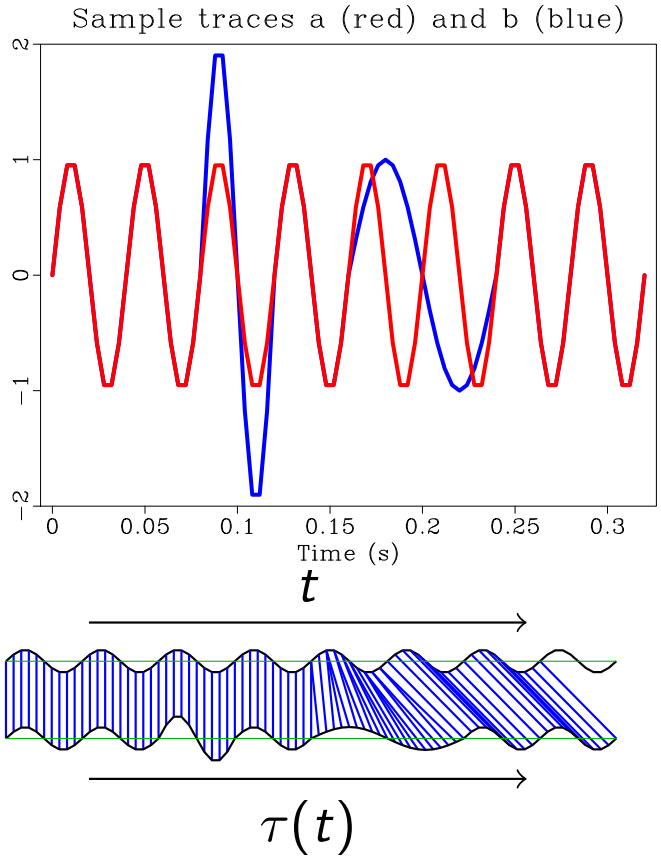


Windowed correlation.



Time shifts — What characterises them?



Define the *temporal warping function* $\tau(t) = t'$.

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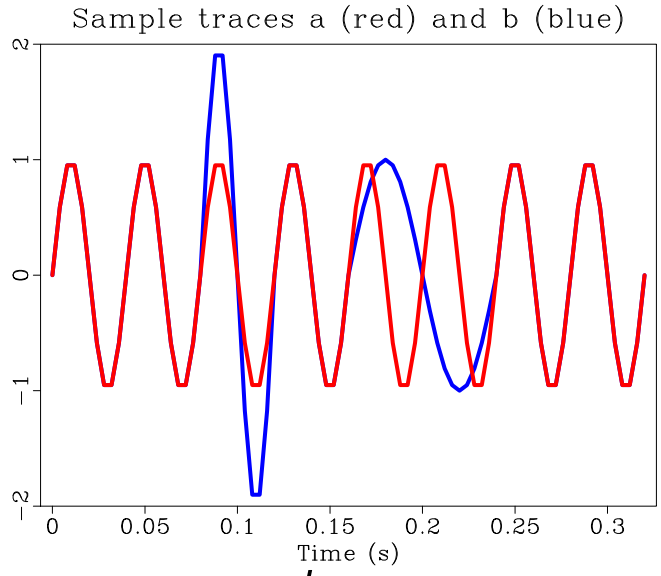

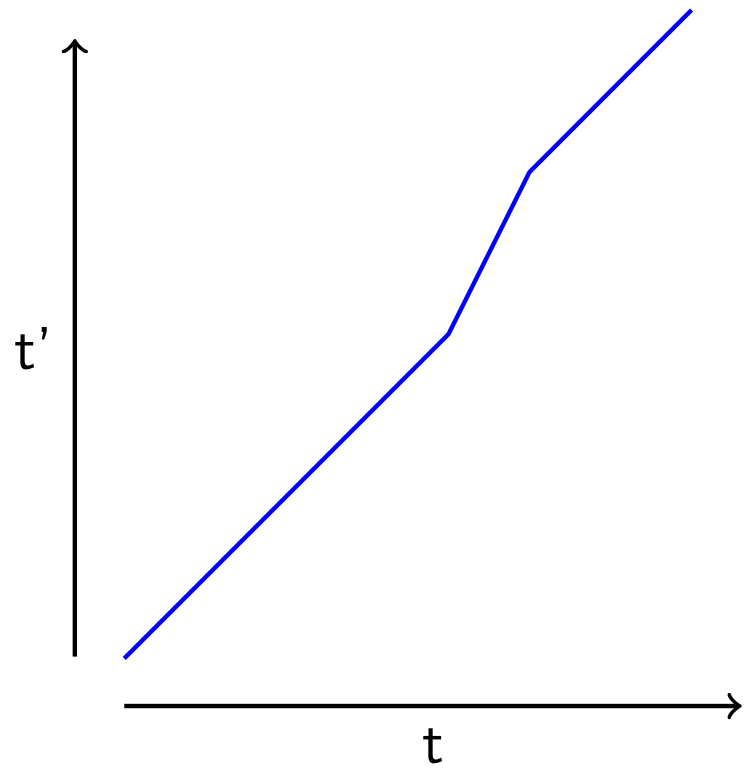

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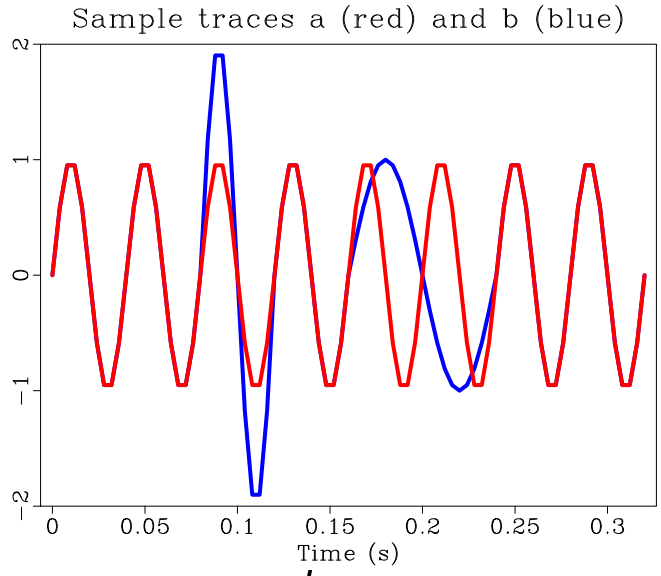
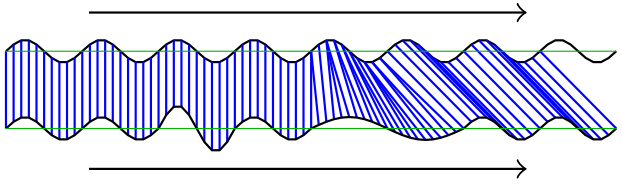


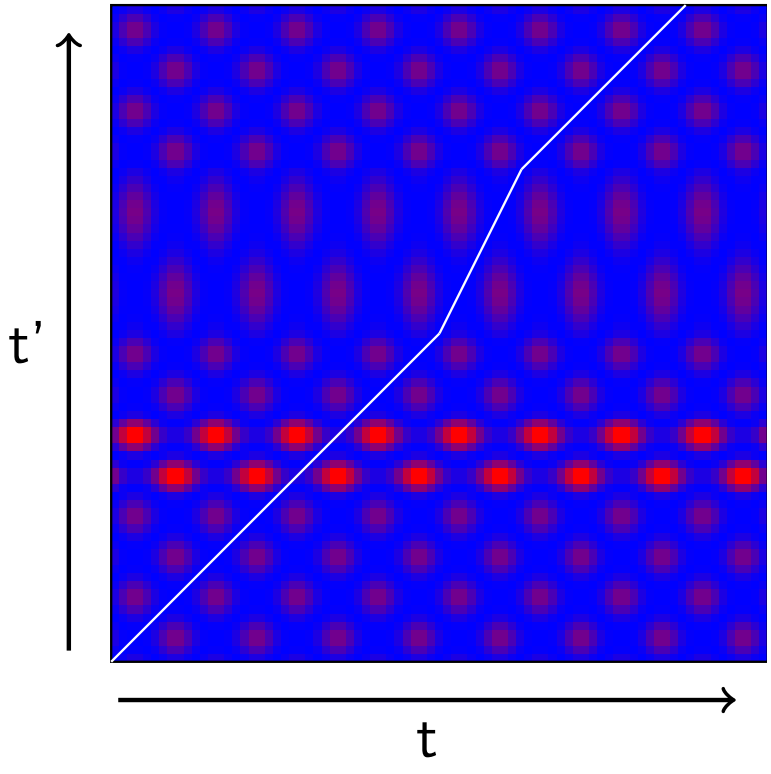
t



$\tau(t)$

Define the *temporal warping function* $\tau(t) = t'$.

$\left(b(t') - a(t)\right)^2$



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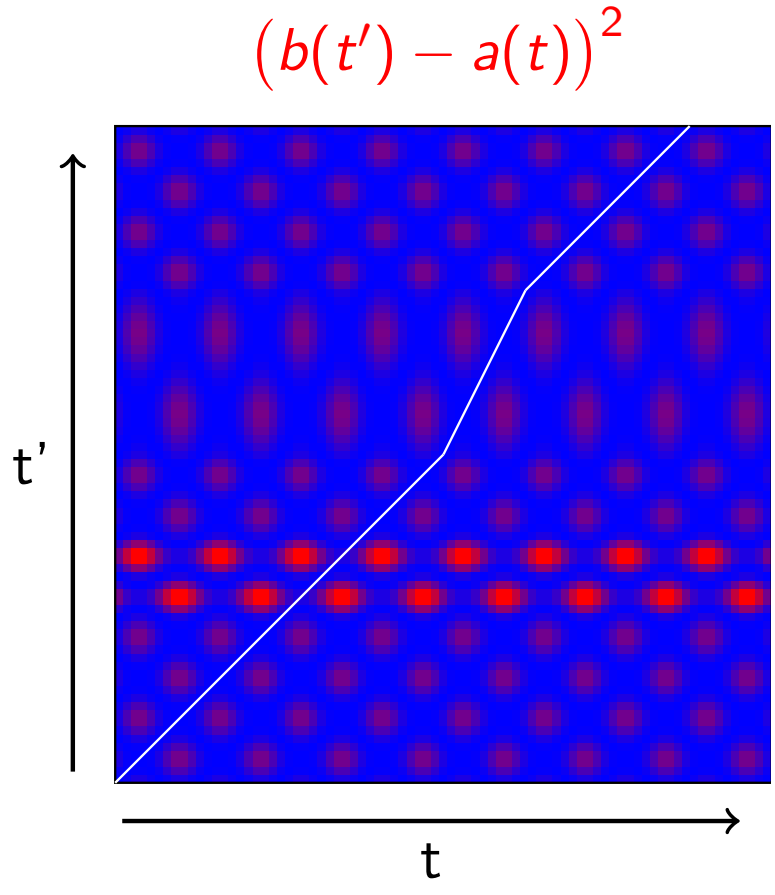
Minimise:

$$\int_0^T \Big(b(\tau(t)) - a(t) \Big)^2$$

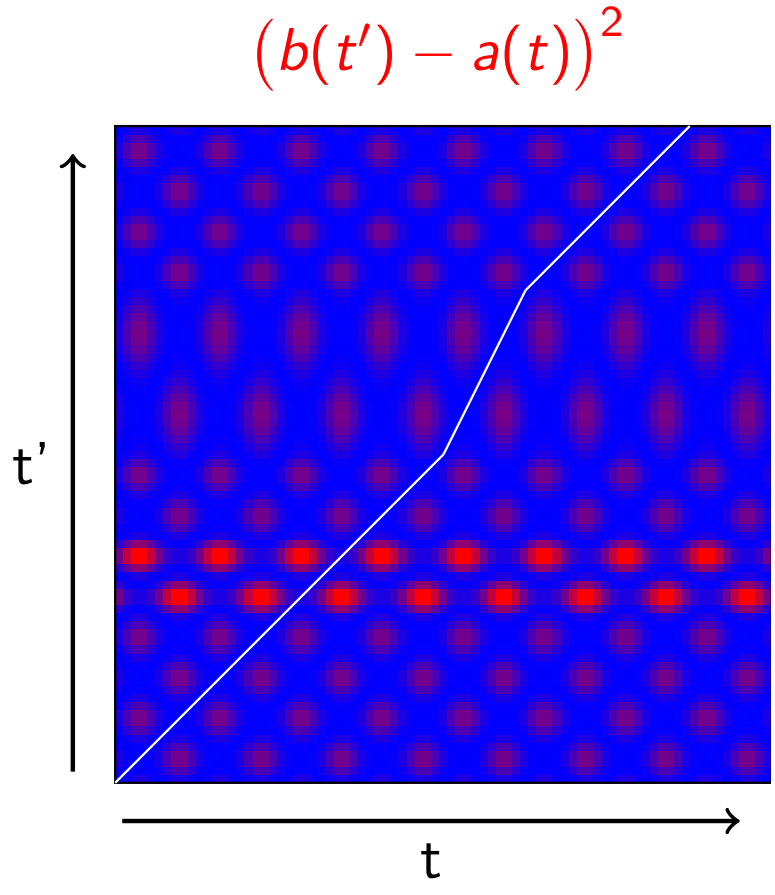


τ is a function ...

Time shifts — What characterises them?

Minimise:

$$\int_0^T \left(b(\tau(t)) - a(t) \right)^2 + \left(\frac{\partial}{\partial t} (\tau(t) - t) \right)^2 \delta t$$



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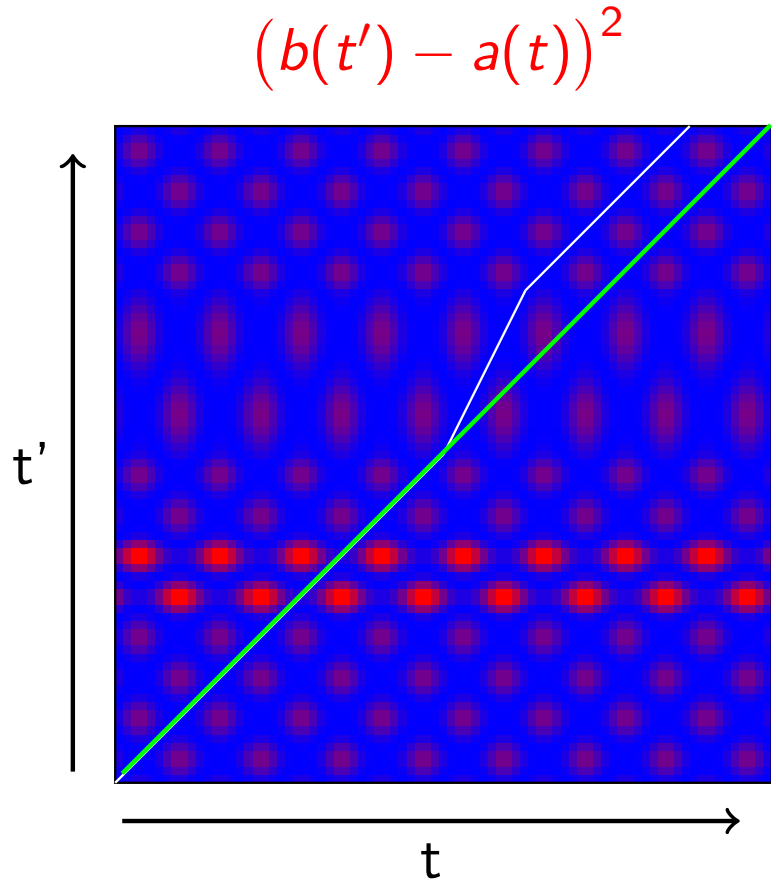

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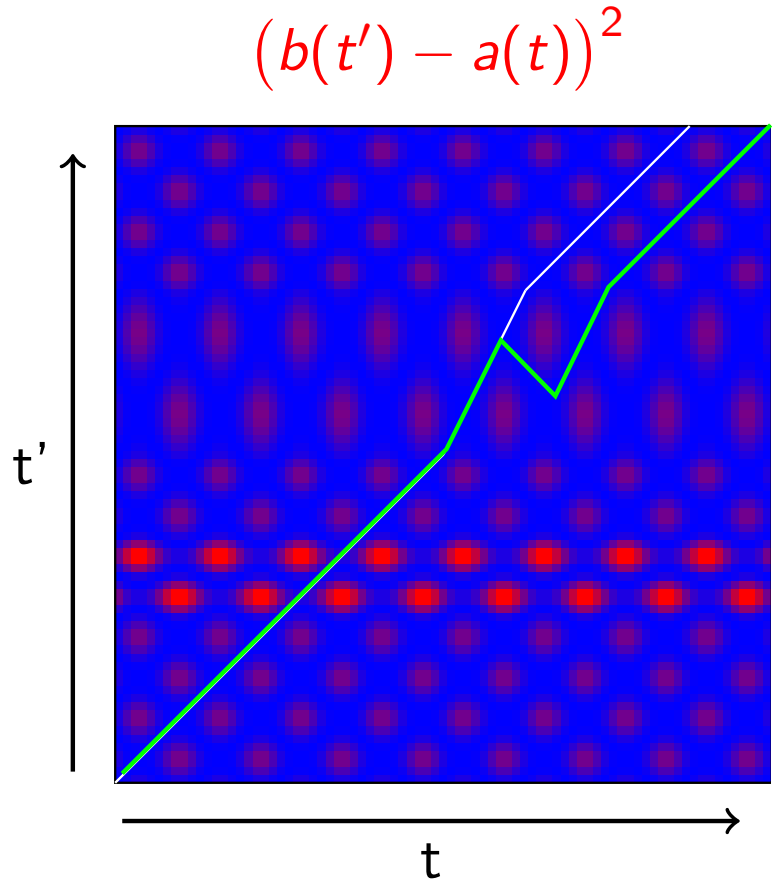

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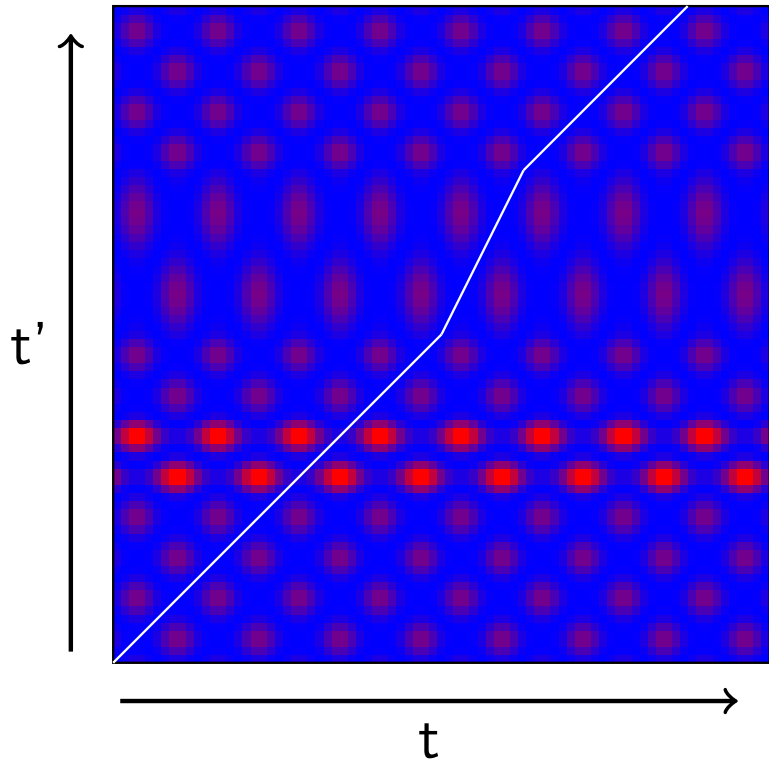

τ is an invertible function ...

Time shifts — What characterises them?

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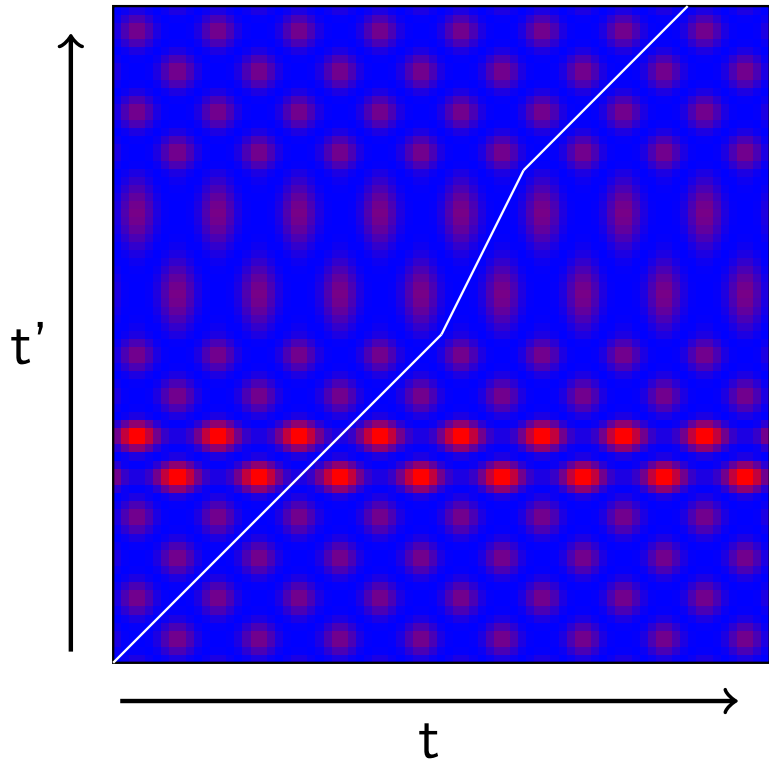

τ is an invertible function ...
and corresponds to a *minimum-cost path*!

Automated time shift estimation — How can it be done?

Cost along line segment:

$$d\big(a(t), b(t')\big)\,\big\|(\Delta t, \Delta t')\big\|$$

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$$\tau_{dtw} = \operatorname{argmin}_{\tau}\left(\int_0^T d\Big(a(t), b\big(\tau(t)\big)\Big) \cdot \left\|\left(1, \frac{\partial}{\partial t}\tau(t)\right)\right\| \delta t\right)$$

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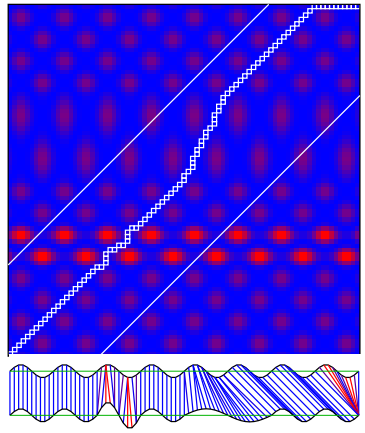

DTW results.

$$\tau_{dtw} = \operatorname{argmin}_{\tau}\left(\int_0^T d\Big(a(t), b\big(\tau(t)\big)\Big)\right.$$

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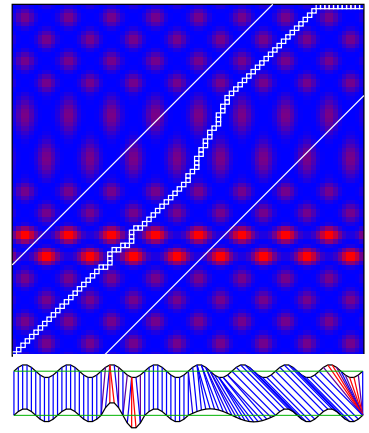

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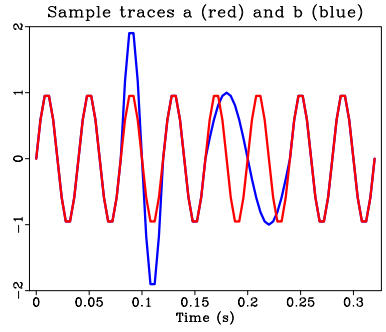

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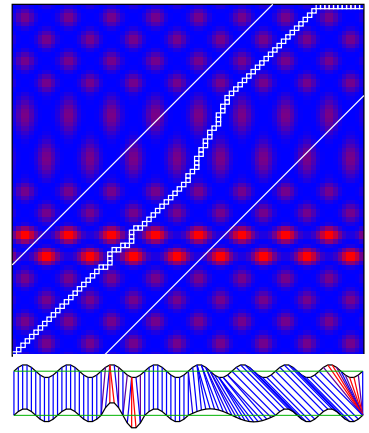
Penalise temporal warping:

$$\alpha\big|\Delta t - \Delta t'\big|, \alpha \in [0, \infty)$$



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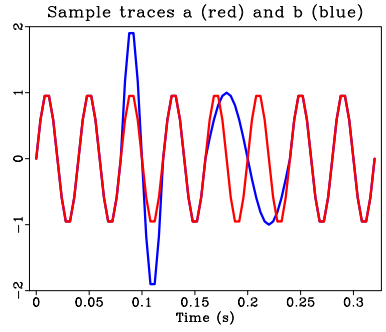

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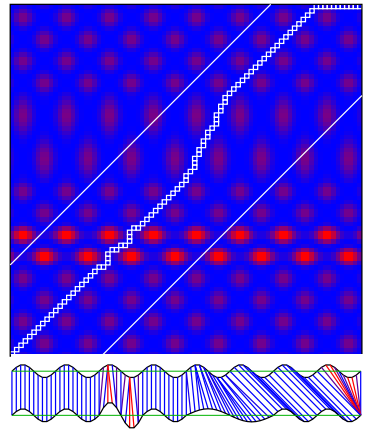
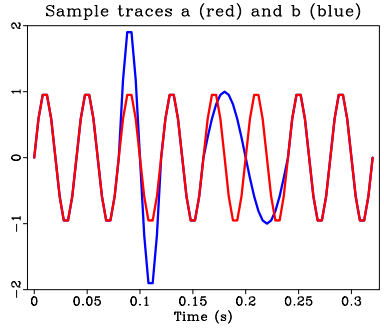


DTW results.

$$\tau_{dtw} = \mathrm{argmin}_{\tau}\left(\int_0^T d\Big(a(t), b\big(\tau(t)\big)\Big)\right.$$

$$\left.\cdot\left\|\left(1, \frac{\partial}{\partial t}\tau(t)\right)\right\| + \alpha\left|1 - \frac{\partial}{\partial t}\tau(t)\right|\delta t\right)$$



Automated time shift estimation — How can it be done?

Cost along line segment:

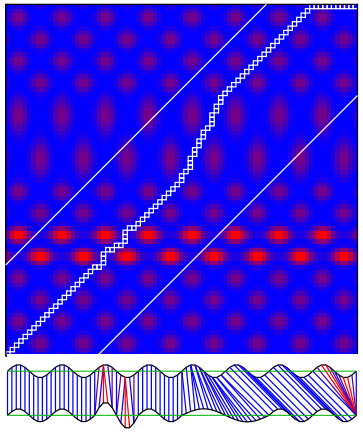
$$d\big(a(t), b(t')\big)\,\big\|(\Delta t, \Delta t')\big\|$$

Penalise temporal warping:

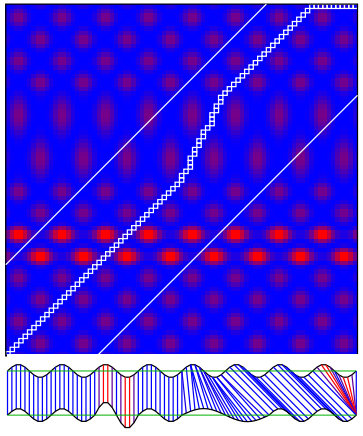
$$\alpha\big|\Delta t - \Delta t'\big|, \alpha \in [0, \infty)$$

$$\tau_{dtw} = \operatorname{argmin}_{\tau}\left(\int_0^T d\Big(a(t), b\big(\tau(t)\big)\Big)\right.$$

$$\left.\cdot\left\|\left(1, \frac{\partial}{\partial t}\tau\,(t)\right)\right\| + \alpha\left|1 - \frac{\partial}{\partial t}\tau(t)\right|\delta t\right)$$



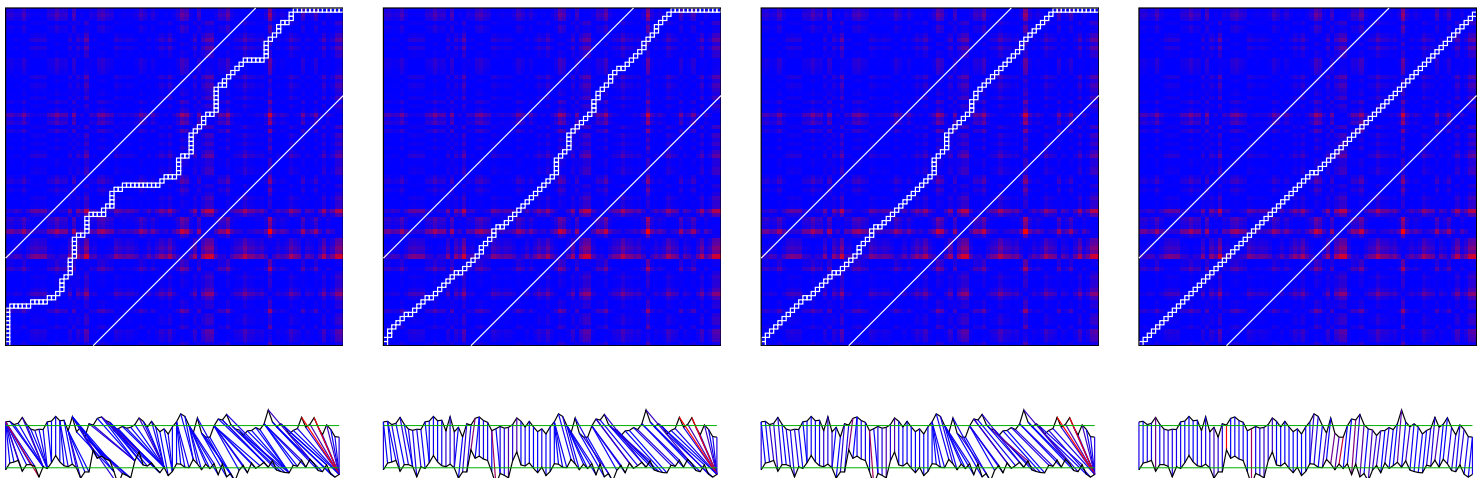
DTW results.



DTW results, $\alpha = 3$.

Automated time shift estimation — How can it be done?

α can also suppress noise: $<$ 125Hz noise, signal-to-noise ratio 1.

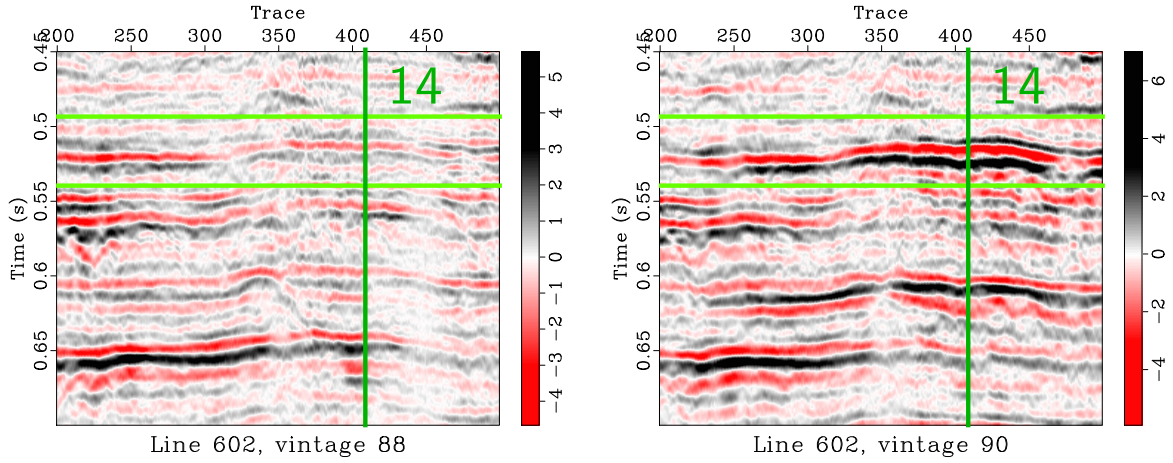


(a) $\alpha = 0$. (b) $\alpha = 1$. (c) $\alpha = 3$. (d) $\alpha = 10$.

Finding the correct α is not generally solved.

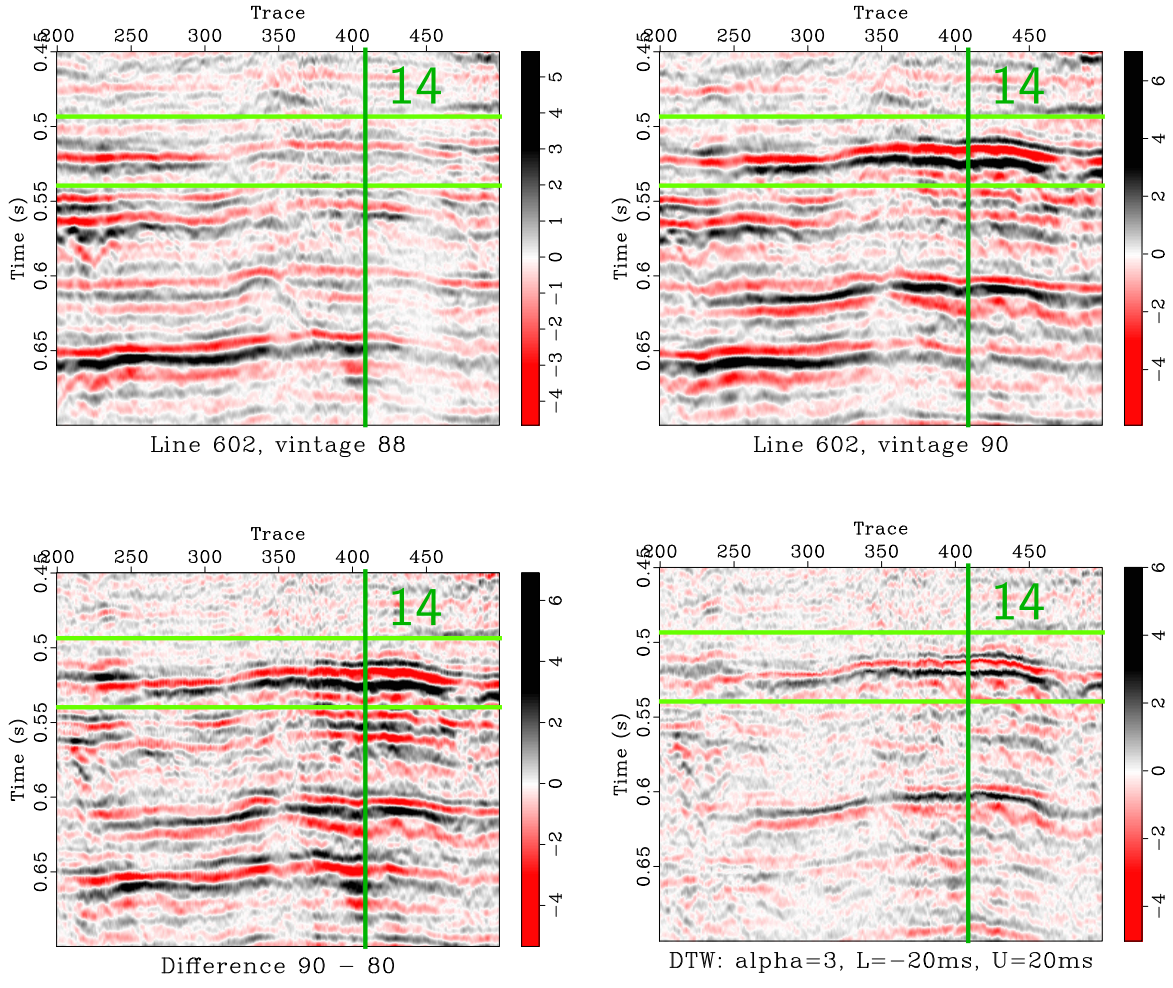
Experimental validation — Time lapse data.

- Confirmed gas leakage into a sand layer at 520ms.



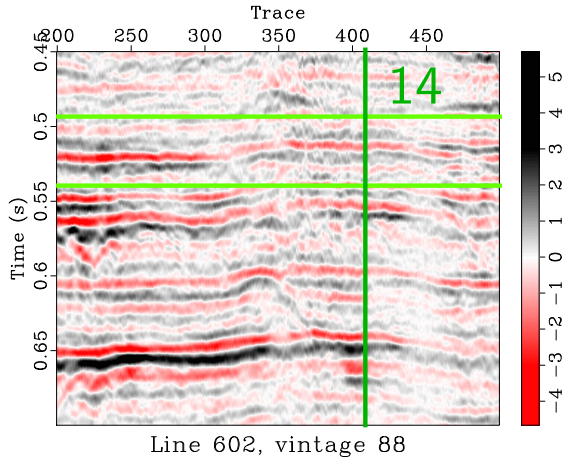
Experimental validation — Time lapse data.

- Confirmed gas leakage into a sand layer at 520ms.
- Amplitude increase between the green lines.

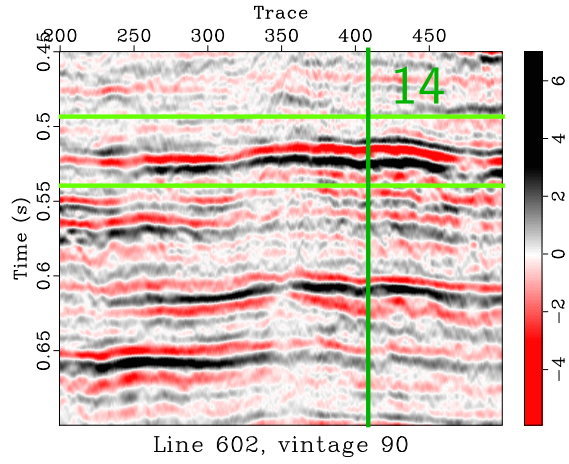


Experimental validation — Time lapse data.

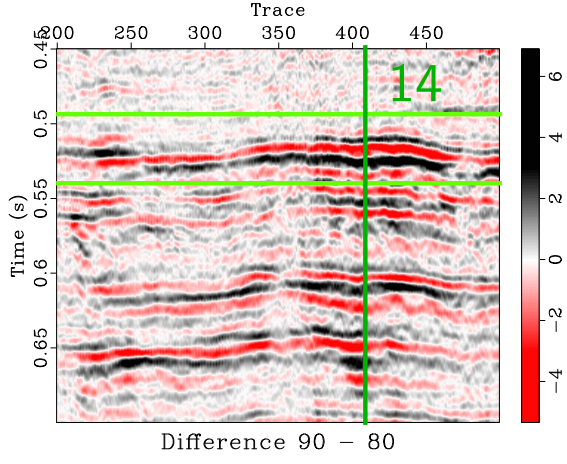
- Confirmed gas leakage into a sand layer at 520ms.
- Amplitude increase between the green lines.
- Time shift increase in the same interval.



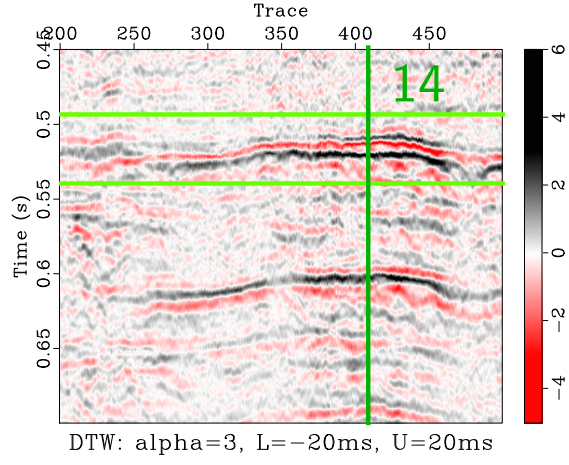
Line 602, vintage 88



Line 602, vintage 90



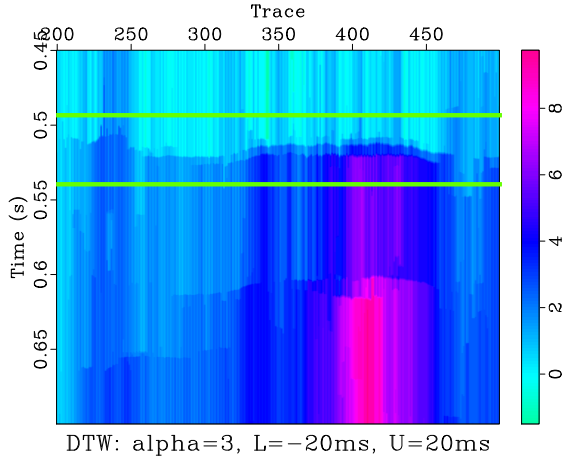
Difference 90 − 80

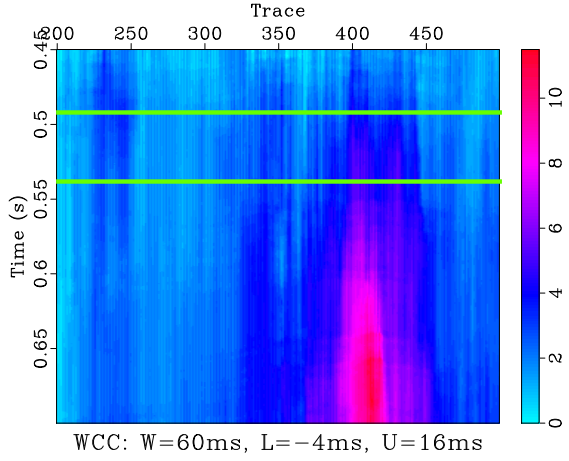


DTW: alpha=3, L=−20ms, U=20ms

Experimental validation — Time lapse data.

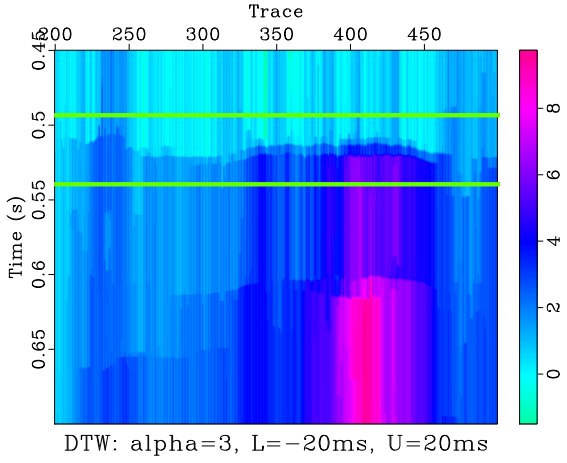
- Main trends similar, but DTW shifts are sharper.

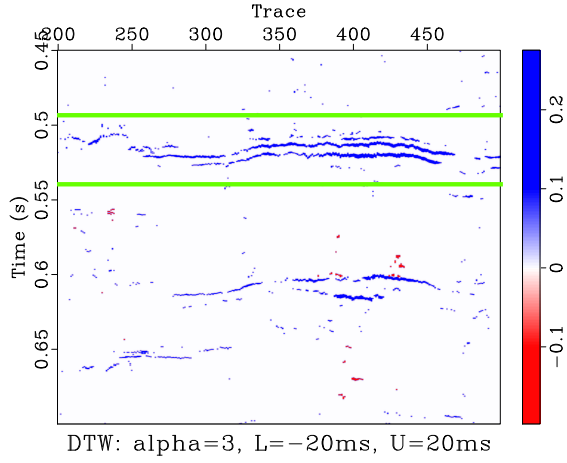


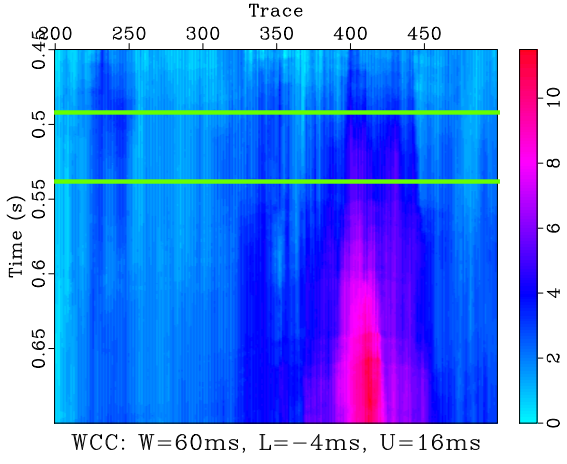


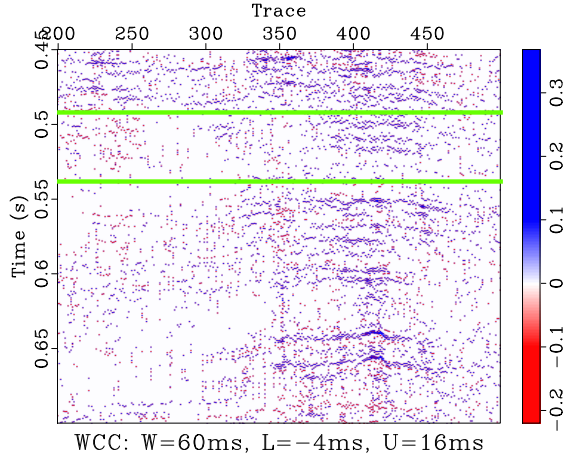
Experimental validation — Time lapse data.

- Main trends similar, but DTW shifts are sharper.
- DTW focuses the time strain according to expectation.



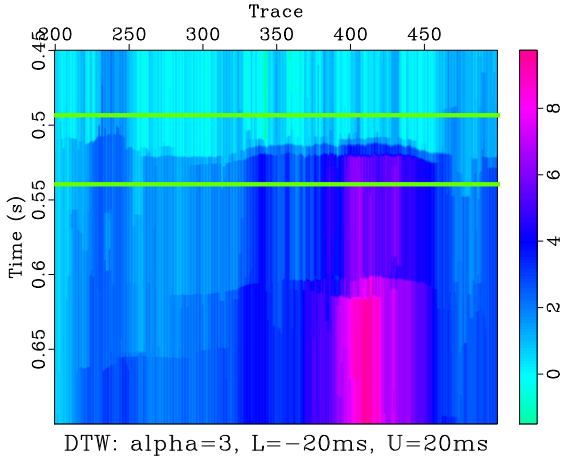


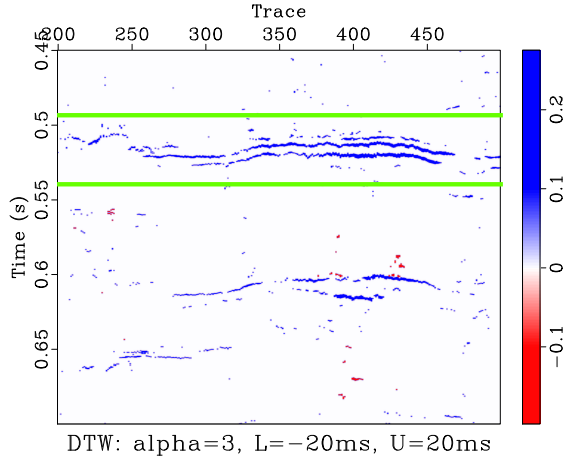


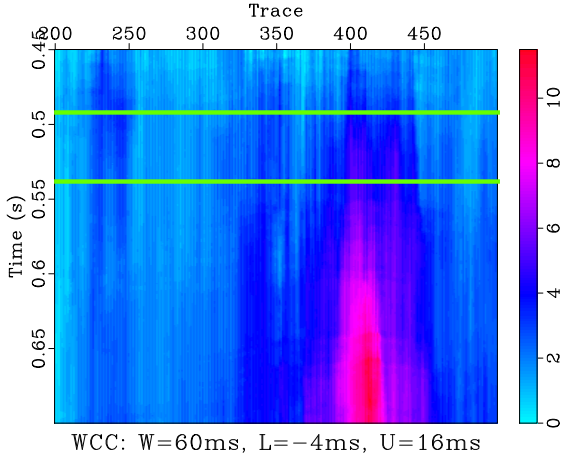


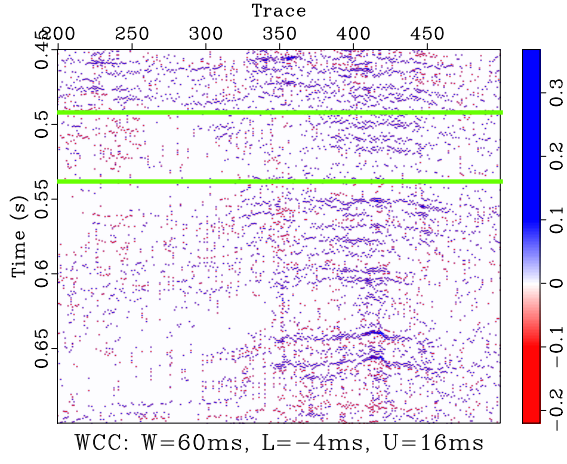
Experimental validation — Time lapse data.

- Main trends similar, but DTW shifts are sharper.
- DTW focuses the time strain according to expectation.
- Horizontal coherence can only be explained by data features.



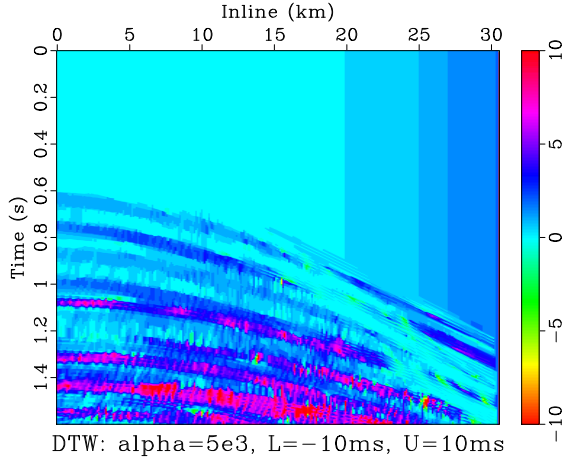


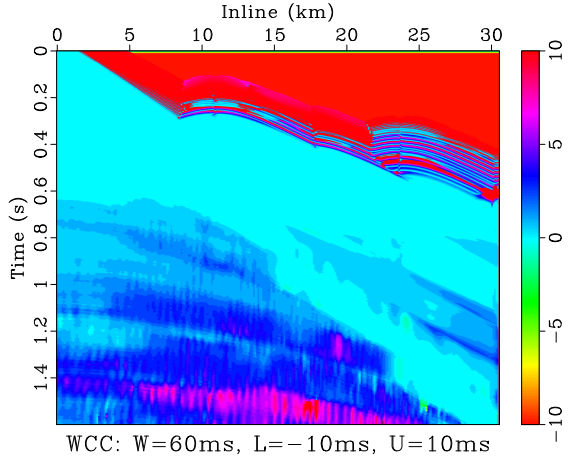




Experimental validation — Comparison with known shifts.

- For a small velocity perturbation, both methods should be correct.





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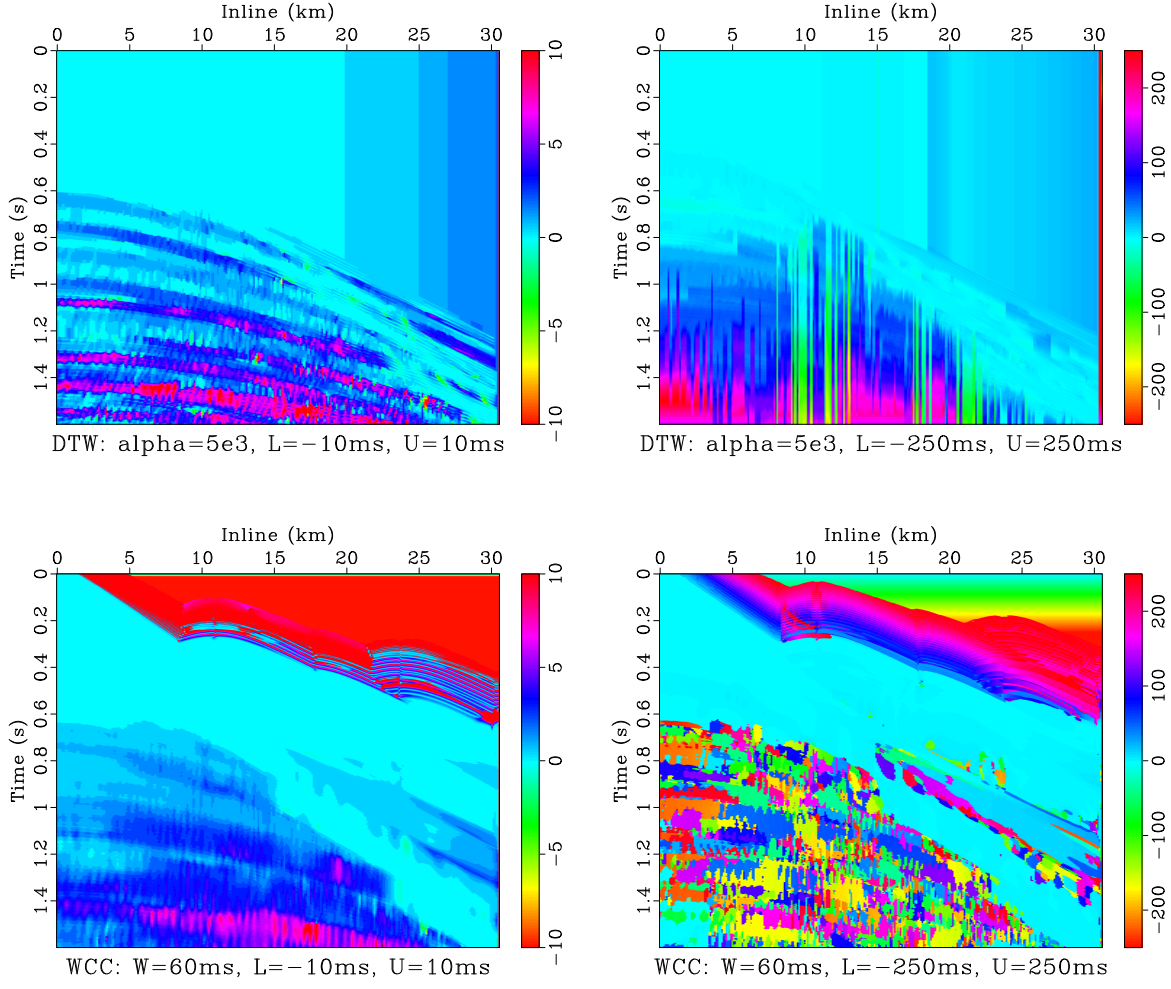
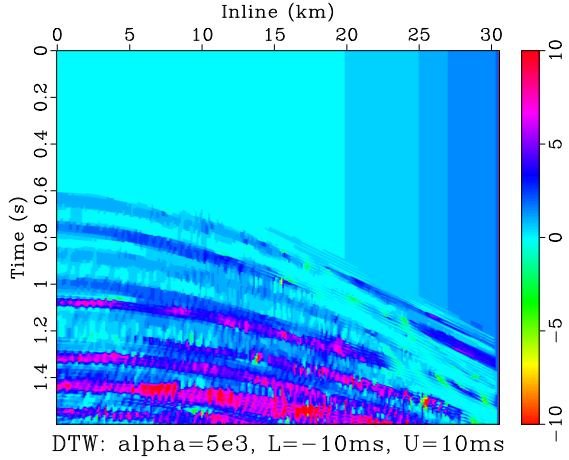
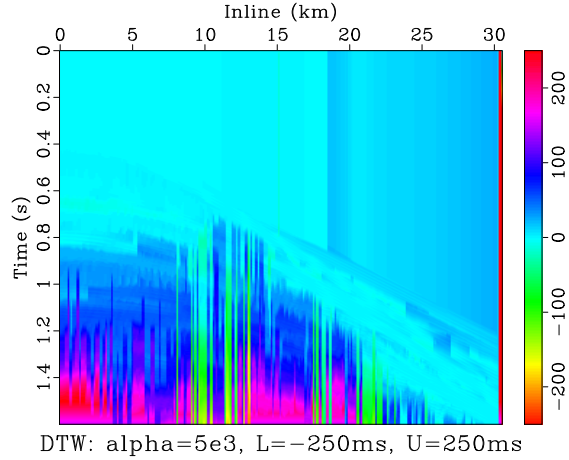
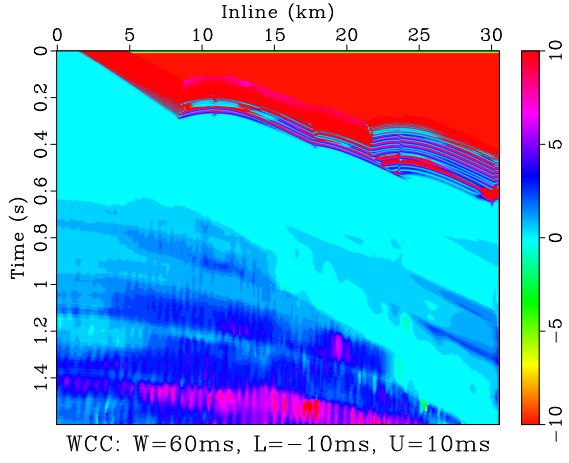


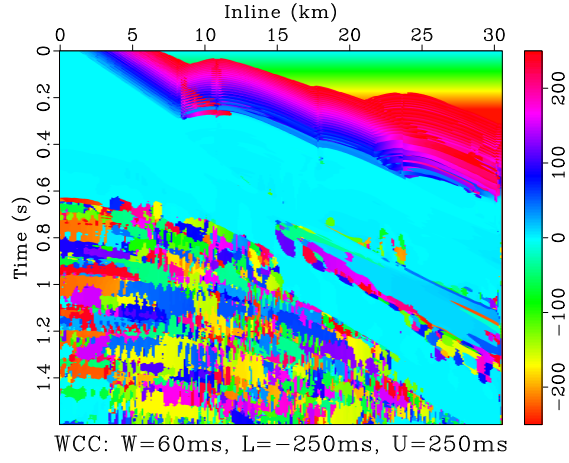
Experimental validation — Comparison with known shifts.

- For a small velocity perturbation, both methods should be correct.
- For the great perturbation, the results disagree totally.
- DTW results similar to each other, continuous and in the right direction!









Summary and remarks.

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Summary and remarks.

- DTW is a minimisation algorithm, that solves the minimisation optimally.
- This gives good resilience against cycle skips.
- DTW also seems to give sharp and precise time shifts.
- Assumptions were that the shifts were invertible, continuous and minimise the alignment mismatch.
- DTW is flexible, and can easily be used in conjunction with other methods.

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