The influence of anisotropy on elastic full-waveform inversion

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Outline

Introduction

- Recently implemented anisotropic (VTI) modeling and FWI.
- Test code on different assumptions used in FWI.
- For synthetic data that are both elastic and anisotropic, investigate quality of inverted V_{P0} model for:
    - Acoustic vs. elastic
    - Isotropic vs. anisotropic
- Try to invert for Thomsen anisotropy parameters ε and δ.

Theory

- In FWI we want to find a parameter model $\mathbf{m}$ that can produce modeled data $\mathbf{u}$ which is close to some measured data $\mathbf{d}$.
- Apply a numerical wave operator that maps $\mathbf{m}$ from the model domain into the data domain:

$$\mathcal{L}(\mathbf{m}) = \mathbf{u}. \tag{1}$$

- Ideally, find an inverse operator to map $\mathbf{d}$ from the data domain to the model domain:

$$\mathbf{m} = \mathcal{L}^{-1}(\mathbf{d}). \tag{2}$$

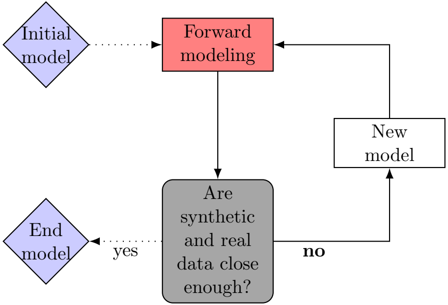
Theory

- Define a misfit functional:

$$\mathcal{F}(\mathbf{m}) = \frac{1}{2}\sum_{j=0}^{n_s}\sum_{i=0}^{n_r} ||\hat{\mathbf{u}}_{i,j}(\mathbf{m}) - \hat{\mathbf{d}}_{i,j}||_2^2. \quad (3)$$

- The solution is an extreme point of $\mathcal{F}(\mathbf{m})$:

$$\mathbf{m}' = \arg\min_{\mathbf{m}} \mathcal{F}(\mathbf{m}). \quad (4)$$



Theory

- Update the model iteratively:

$$\mathbf{m}_{k+1} = \mathbf{m}_k - \alpha_k \mathbf{H}_k^{-1} \delta \mathbf{m}_k. \tag{5}$$

- Hessian matrix contains second derivatives of the misfit functional
  - Approximated from previous gradients (L-BFGS)
- Gradients are found via the adjoint method, Mora (1987).

$$\delta \hat{\mathbf{m}}(\mathbf{x}) = \sum_{n_s} \int \mathrm{d}t \sum_{n_r} \frac{\partial u_i(\mathbf{x_S}, \mathbf{x_R}, t)}{\partial \mathbf{m}(\mathbf{x})} \delta u_i(\mathbf{x_S}, \mathbf{x_R}, t). \tag{6}$$

$$\delta u_i(\mathbf{x_S}, \mathbf{x_R}, t) = \int_V \mathrm{d}V \frac{\partial u_i(\mathbf{x_S}, \mathbf{x_R}, t)}{\partial \mathbf{m}(\mathbf{x})} \delta \mathbf{m}(\mathbf{x}). \tag{7}$$

Gradients

$$\delta\rho = -\sum_{n_s}\int \mathrm{d}t\dot{u}_j\dot{\Psi}_j,$$

$$\delta c_{11} = -\sum_{n_s}\int \mathrm{d}t(u_{1,1}+u_{2,2})(\Psi_{1,1}+\Psi_{2,2}),$$

$$\delta c_{33} = -\sum_{n_s}\int \mathrm{d}t u_{3,3}\Psi_{3,3},$$

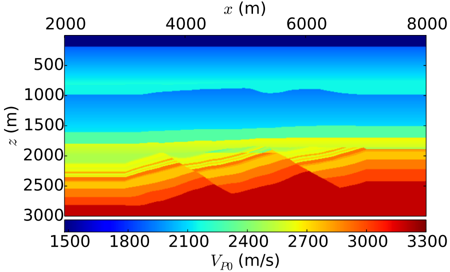
$$\delta c_{13} = -\sum_{n_s}\int \mathrm{d}t\Big[\Psi_{3,3}(u_{1,1}+u_{2,2})+(\Psi_{1,1}+\Psi_{2,2})u_{3,3}\Big],$$

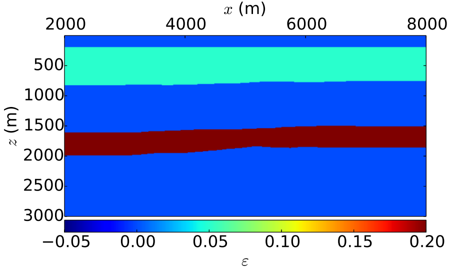
$$\delta c_{44} = -\sum_{n_s}\int \mathrm{d}t\Big[(\Psi_{3,1}+\Psi_{1,3})(u_{3,1}+u_{1,3}) + (\Psi_{3,2}+\Psi_{2,3})(u_{3,2}+u_{2,3})\Big],$$

$$\delta c_{66} = -\sum_{n_s}\int \mathrm{d}t\Big[(\Psi_{2,1}+\Psi_{1,2})(u_{2,1}+u_{1,2}) - 2(\Psi_{2,2}u_{1,1}+\Psi_{1,1}u_{2,2})\Big].$$

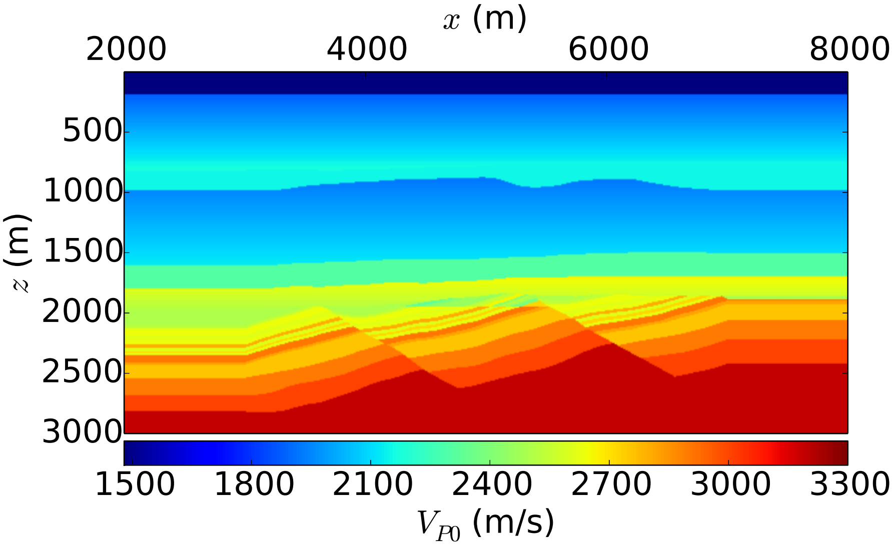
Model

- Synthetic model representative of the Gullfaks field
- 10 km long, 3 km deep
- 1001×300 grid points
- Total of 101 shots and 1001 receivers
- Source: 5 Hz Ricker wavelet
- Receivers: Pressure
- Gradient muted in the water layer

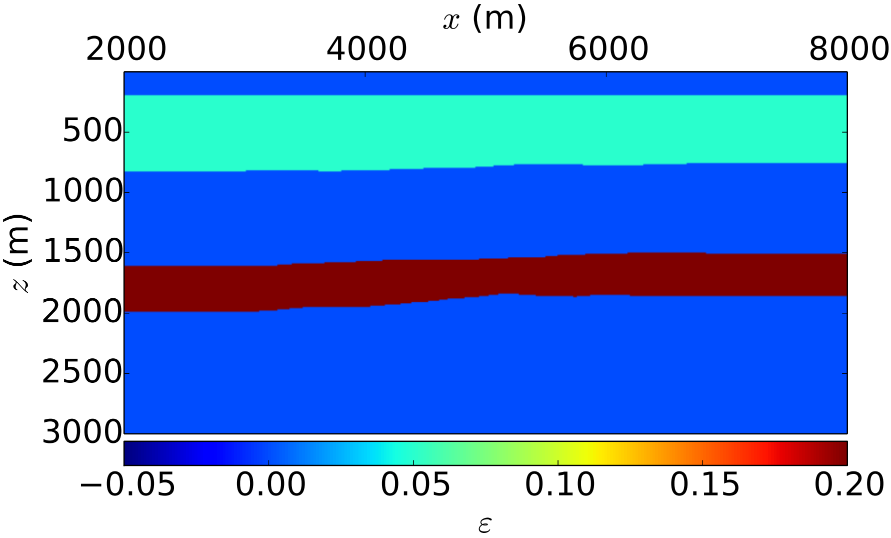




Model

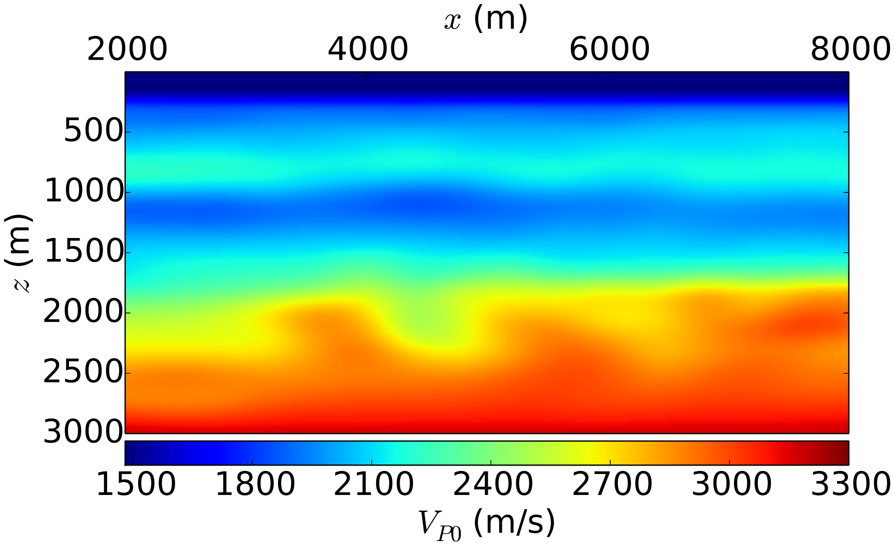

x (m)
2000
4000
6000
8000
z (m)
500
1000
1500
2000
2500
3000
1500
1800
2100
2400
2700
3000
3300
V_{P0} (m/s)

Model

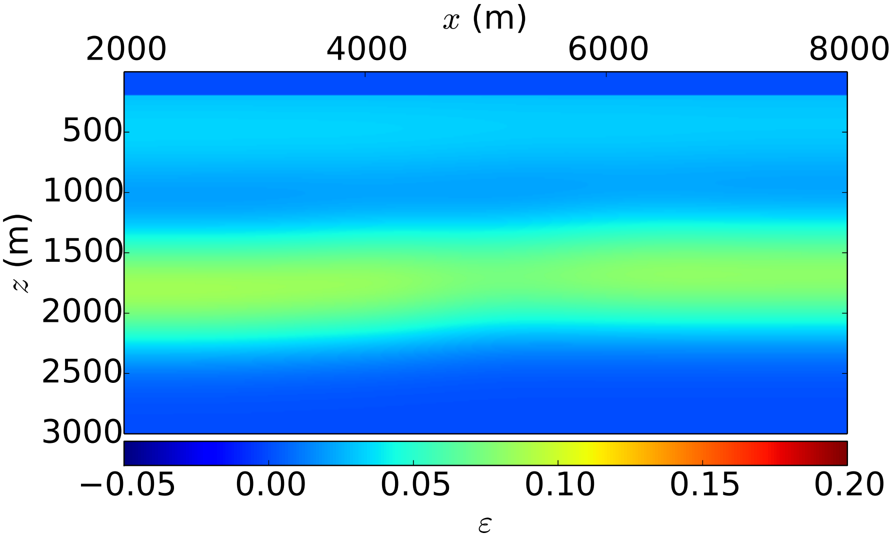


x (m)
2000
4000
6000
8000
z (m)
500
1000
1500
2000
2500
3000
−0.05
0.00
0.05
0.10
0.15
0.20
ε

Starting model



Starting model



Results

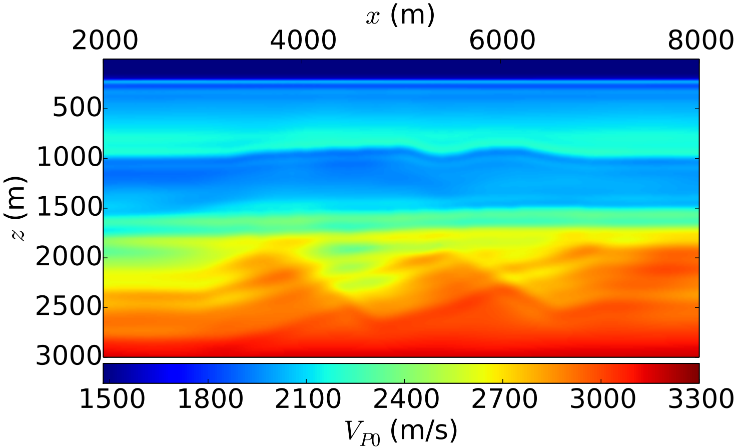


Figure : Inverted model for V_{P0} with exact ε and δ, elastic.

Results

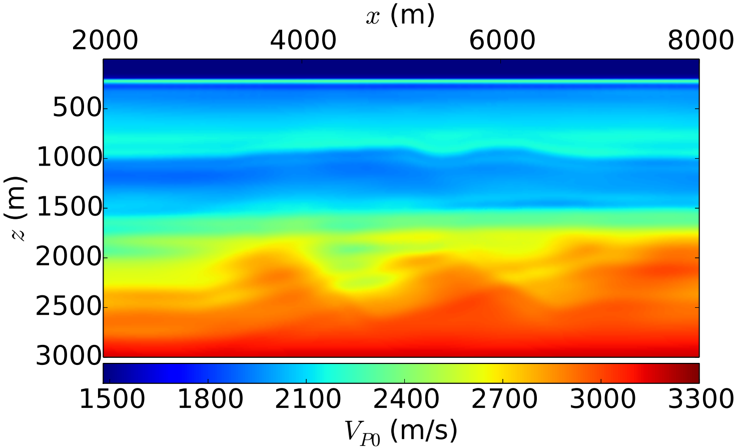
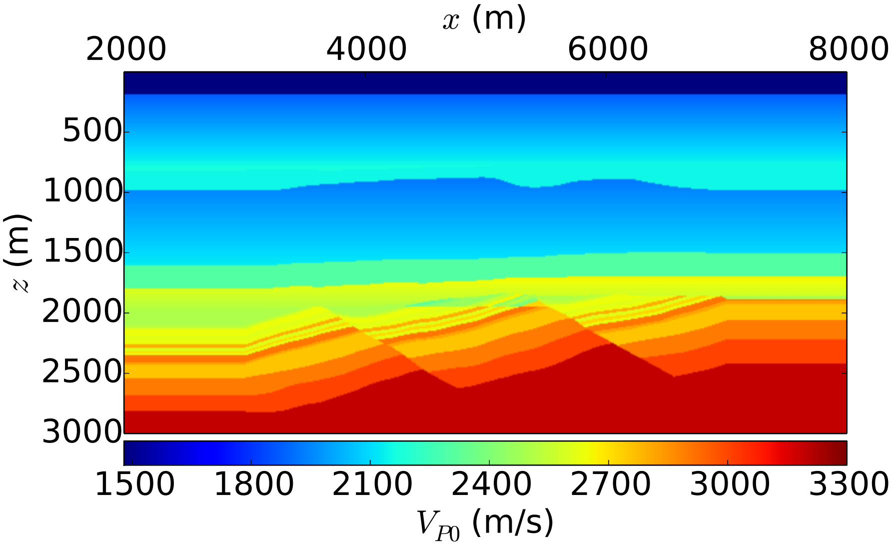


Figure : Inverted model for V_{P0} with exact ε and δ, acoustic.

True model



x (m)
2000
4000
6000
8000
z (m)
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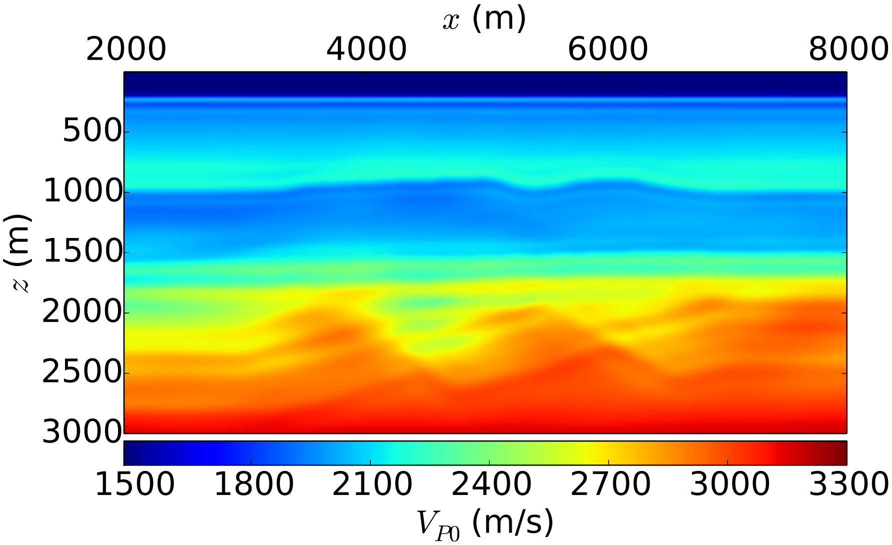
1000
1500
2000
2500
3000
1500
1800
2100
2400
2700
3000
3300
V_{P0} (m/s)

Elastic inversion



Results

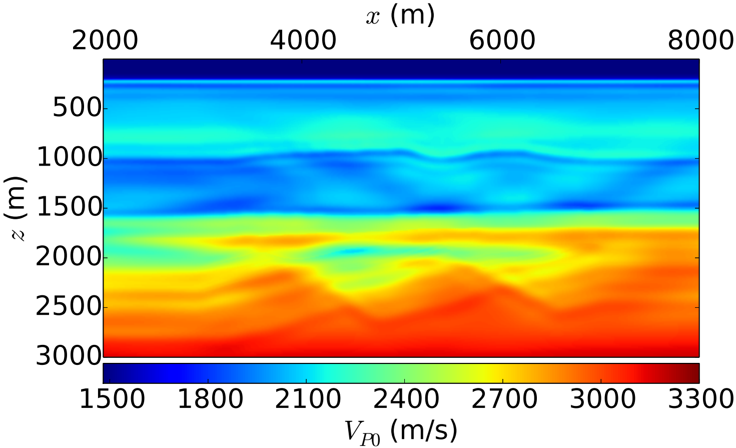


Figure : Inverted model for V_{P0} with smooth ε and δ

Results

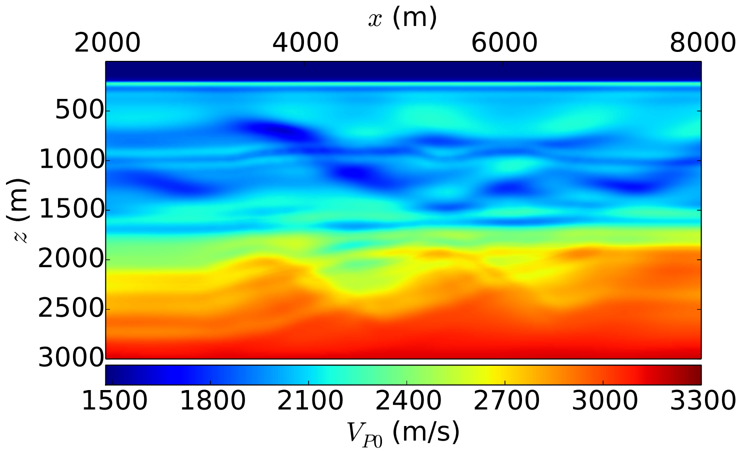


Figure : Inverted model for V_{P0} with $\varepsilon = \delta = 0$, elastic

Results

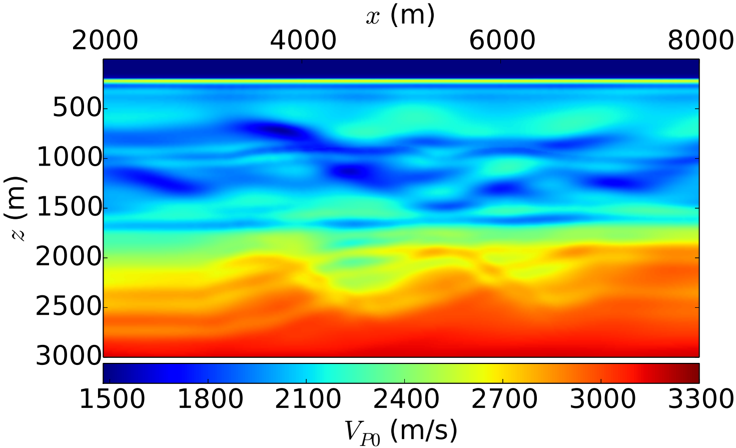


Figure : Inverted model for V_{P0} with $\varepsilon = \delta = 0$, acoustic.

Results

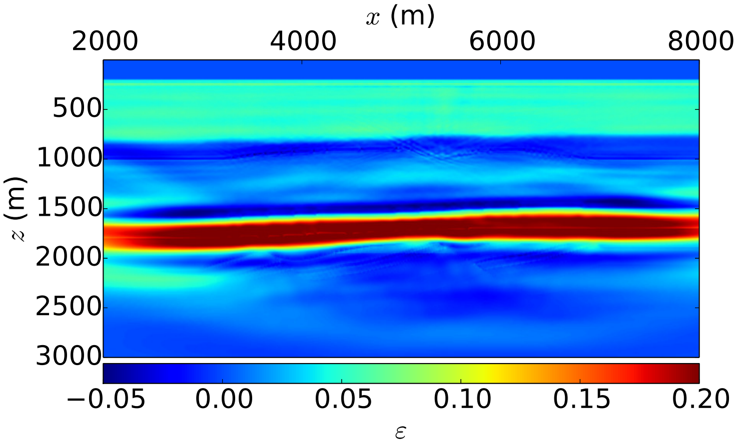
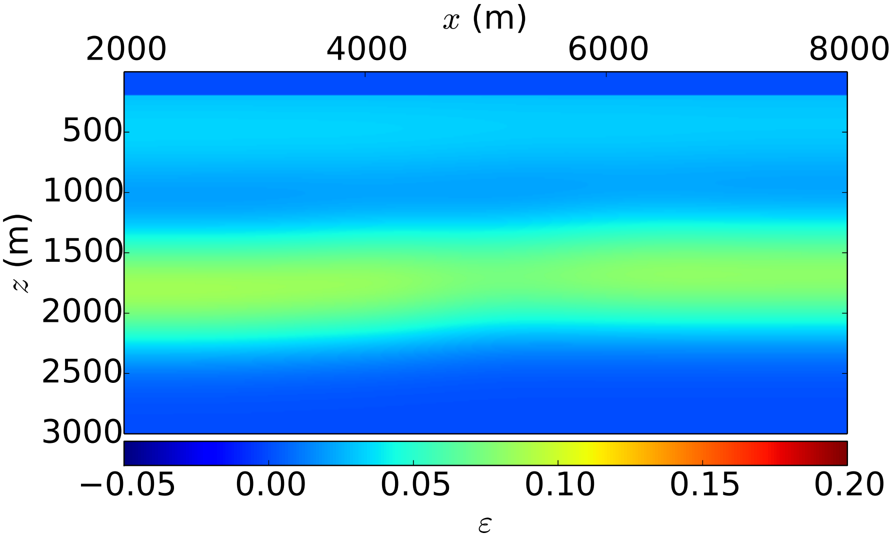
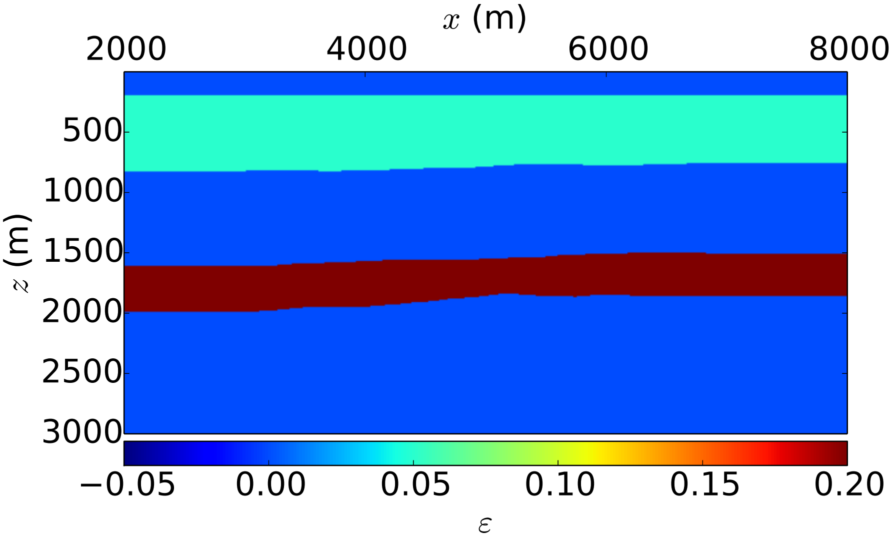


Figure : Inverted model for ε.

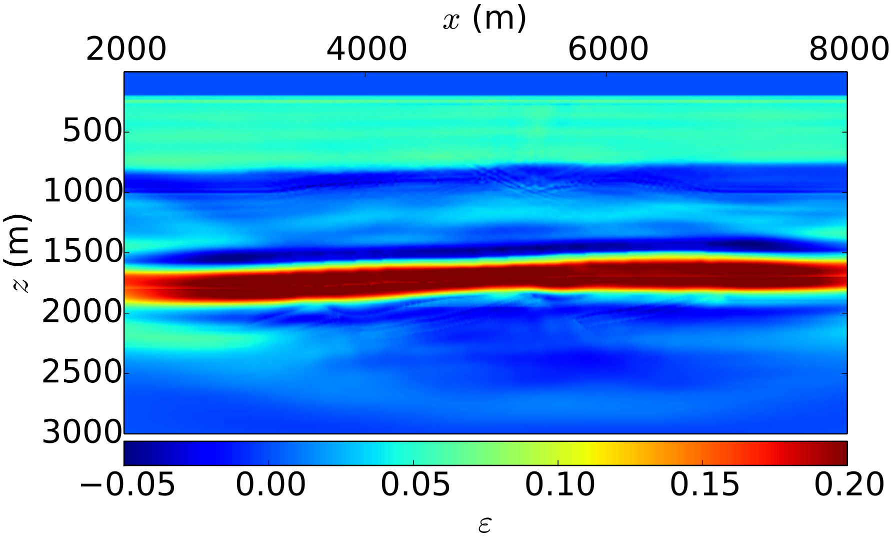
Starting model



True model


x (m)
2000
4000
6000
8000
z (m)
500
1000
1500
2000
2500
3000
−0.05
0.00
0.05
0.10
0.15
0.20
ε

Inverted model



Results

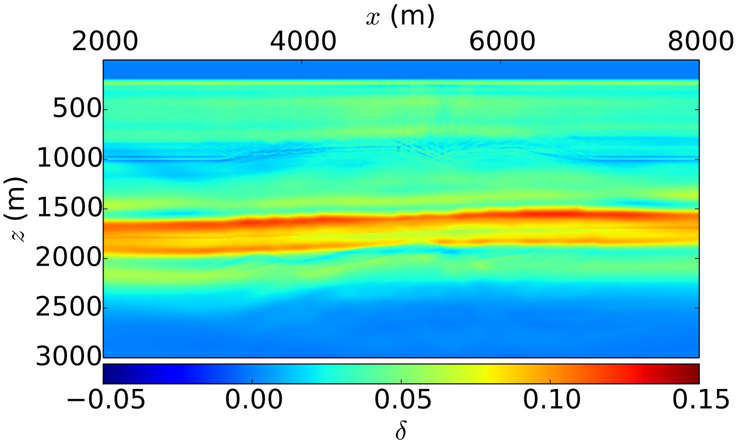


Figure : Inverted model for δ.

Conclusions

- Four different inversion assumptions applied to an elastic, anisotropic dataset.
- Acoustic approximation holds, due to long offset data.
- Anisotropy cannot be completely neglected.
- A perfect anisotropy model is not needed, but some knowledge is necessary.
- Inverting for ε and δ is in principle possible.

Acknowledgments

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