# Estimation of thickness and velocity changes of injected carbon dioxide layers from prestack time-lapse seismic data 

Amir Ghaderi ${ }^{1}$ and Martin Landrø ${ }^{1}$


#### Abstract

In this study, we bring together the two main categories of time-lapse seismic analysis - amplitude analysis and time-shift analysis - to estimate simultaneously the changes in thickness and velocity of a 4D seismic anomaly. The methodology is applied to time-lapse seismic monitoring of carbon dioxide $\left(\mathrm{CO}_{2}\right)$ storage at Sleipner field, Norway, that shows significant 4D effects. The 4D anomalies resulting from $\mathrm{CO}_{2}$ injection appear as a multilayer reflection pattern within the relatively shallow Utsira Sand. This multireflective appearance within the sand layer is interpreted as $\mathrm{CO}_{2}$ layers trapped below thin shale layers. Because most of the $\mathrm{CO}_{2}$ layers are believed to be thin ( $0-20 \mathrm{~m}$ ), the interference between top and base of these layers needs to be taken into account in 4D seismic analysis. By studying the reflected event from a horizon below the Utsira Sand, we estimate 4D traveltime shifts caused by the presence of the $\mathrm{CO}_{2}$ layer above this


horizon. We then combine these traveltime shifts with measured amplitude changes for the top and base of the $\mathrm{CO}_{2}$ layer to estimate velocity and thickness changes for the thin $\mathrm{CO}_{2}$ layer. In 1999, after three years of injection, the most likely velocity change was around $200 \mathrm{~m} / \mathrm{s}$ and the thickness of the $\mathrm{CO}_{2}$ layer was around 15 m . In 2001, the corresponding velocity change and thickness estimates were $400 \mathrm{~m} / \mathrm{s}$ and 15 m , respectively. Finally, in 2002, the most likely velocity change was $500 \mathrm{~m} / \mathrm{s}$ and the thickness of the $\mathrm{CO}_{2}$ layer was 15 m . It is not straightforward to apply this method to a stack of $\mathrm{CO}_{2}$ layers because 4D time shifts below the Utsira Sand only provide information about the average time shift for all layers. The amplitude information for each individual $\mathrm{CO}_{2}$ layer cannot be resolved without knowing the velocity change within each layer. However, our result from a single $\mathrm{CO}_{2}$ layer may be used to constrain the velocity changes for the multilayer $\mathrm{CO}_{2}$ case.

## INTRODUCTION

Time-lapse seismic analysis can be divided into two main categories: amplitude analysis and time-shift analysis. These complementary methods often can be used to increase the knowledge of intrareservoir changes (Landrø et al., 2001). For compacting reservoirs, time shift has been the major technique (Guilbot and Smith, 2002; Landrø and Stammeijer, 2004; Hatchell and Bourne, 2005) for qualitative and quantitative 4 D analysis. When the same velocities are used for the base and monitor surveys, offset-dependent correction terms are necessary for mid- and far-offset stacks, as discussed by Landrø and Janssen (2002) and Landrø and Stammeijer (2004).

If an accurate velocity analysis is performed separately for the base and monitor surveys (which might be necessary if the timelapse velocity changes are significant), then this correction term is unnecessary. However, as shown by Kvam and Landrø (2005), it is
often difficult to do accurate time-lapse velocity analysis, and then one must take into account the uncertainty in the velocity analysis. Hence, in the presence of the correction term, the poststack traveltime shift cannot be translated directly into velocity changes. Landrø and Stammeijer (2004), assuming zero compaction, and Ghaderi and Landrø (2005) show for prestack seismic data that

$$
\begin{equation*}
\frac{\Delta t}{t} \approx-\frac{\Delta V}{V} \tag{1}
\end{equation*}
$$

where $t$ is the seismic two-way time thickness of the layer and $v$ is the velocity of the layer. In equation 1 it is assumed that $\Delta V \ll V$ and that the change in the incidence angle $\theta$ caused by $\mathrm{CO}_{2}$ injection is negligible. Also note that equation 1 is valid for all offsets, but the assumption that $\Delta \theta$ is small is, of course, less valid for higher offsets.

[^0]So far we have assumed that the velocity anomaly is constant within the offset span used for the analysis. In the Sleipner $\mathrm{CO}_{2}$ data set, the typical lateral extension of the $\mathrm{CO}_{2}$ anomaly is of the order of $750-1500 \mathrm{~m}$. The maximum offset available for the prestack data is 1668 m . This means that edge effects will play an important role for the long-offset time-shift analysis. Hence, a correction formula is

Figure 1. Schematic of $\mathrm{CO}_{2}$ injection in the Utsira Formation at Sleipner (courtesy of Statoil).


Figure 2. A well log from the Utsira Sand, showing the gamma-ray measurements. The gamma-ray values for the cap rock typically are higher than those for the sand body within Utsira. Notice the gammaray peaks within the Utsira Sand correspond to thin shale beds in an otherwise homogeneous thick sand layer. The thin shale layers are assumed to have typical thicknesses of 1-2 m .
needed that can be applied to long-offset data. For this purpose, it is possible to modify the result by Røste et al. (2006). However, in our data example from the Sleipner $\mathrm{CO}_{2}$ project, we use small-offset prestack seismic data, corresponding to a single offset of 318 m . Thus, the correction formula by Røste et al. (2006) is assumed to have negligible effect.

Seismic time shifts usually are estimated by determining the lag that corresponds to the peak in the crosscorrelation function between the two signal traces under consideration. Another method is to pick the traveltime for the maximum amplitude peak of a well-defined seismic event. In comparing the two methods, Landrø et al. (2001) show that the simple pick method is closer to the exact time shifts than the windowed crosscorrelation, as long as the signal-to-noise ratio is larger than one-fourth. We have found that in the case of Utsira 4D seismic analysis, there is a very good correspondence between the two methods. Hence, for the subsequent analysis, we use simple picking.

The 4D amplitude analysis is the most common method of analyzing time-lapse seismic data (Landrø et al., 1999; Koster et al., 2000). Differencing and various comparison methods are used to analyze and interpret 4D seismic changes. For thin reservoir sections, the 4D amplitude signal is enhanced by tuning effects (Landrø et al., 1999). When thin $\mathrm{CO}_{2}$ layers are injected into a relatively thick sand body, the thickness and velocity of the $\mathrm{CO}_{2}$ layers are unknown. Therefore, we need two independent relations to determine the two unknown parameters. Here, we compare two different methods - one combining amplitude and traveltime analysis and the other combining traveltime and rock physics.

## Background for injecting $\mathrm{CO}_{2}$ in the Utsira Formation

Concerns about global warming have initiated worldwide investigation into $\mathrm{CO}_{2}$ disposal. A promising geologic setting for storage is saline rock formations. The Sleipner field in the Norwegian North Sea, operated by StatoilHydro, is the site of a large-scale $\mathrm{CO}_{2}$ disposal project (Eiken et al., 2000). At Sleipner, $\mathrm{CO}_{2}$ is injected into a shallow sand formation (Utsira) 1012 m below sea level (Figure 1). The sand body is about 210 m thick around the injection point and contains some thin shale layers, seen in a well log from Utsira (Figure 2). The formation is sealed at the top by thick shale layers. Below the sand formation, several mud diapirs can be interpreted.

The $\mathrm{CO}_{2}$ is injected at a supercritical state close to the bottom of the formation. It rises because of buoyancy effects until reaching various flow barriers such as shale layers within the sand and the top seal shale. The shale layer above the Utsira Sand is thick and extends laterally throughout the formation. It is therefore reasonable to assume that the injected $\mathrm{CO}_{2}$ will form a plume consisting of thin $\mathrm{CO}_{2}$ layers (lenses) below thin, heterogeneous shale layers within the thick sand body (Lindeberg et al., 2001; Arts et al., 2004). In 1999, 2.35 million tons of $\mathrm{CO}_{2}$ were injected into the Utsira Formation; by 2001, 4.26 million tons of $\mathrm{CO}_{2}$ had been injected. In 2002, the total injection of $\mathrm{CO}_{2}$ was up to 4.97 million tons. A 3D seismic data set was acquired in 1994 prior to $\mathrm{CO}_{2}$ injection, and time-lapse seismic data were acquired in 1999, 2001, 2002, 2004, and 2006.

Several studies have been published using stacked seismic data from the 1994, 1999, 2001, and 2002 surveys (Arts et al., 2004; Chadwick et al., 2004). Ghaderi and Landrø (2005) conducted a preliminary study on estimating velocity changes based on the prestack seismic data vintages from 1994 and 1999 for the Utsira Formation. In the present work, we use prestack seismic data from the base and the first three monitor surveys.

The acquisition parameters for the various surveys at Sleipner have been repeated to a high degree (Eiken et al., 2000). However, there are some changes in shotpoint interval (from 18.75 m in 1994 to 12.5 m in 1999). Because of the severe velocity reduction caused by the presence of $\mathrm{CO}_{2}$, different velocity fields had to be used for stacking and migrating the seismic baseline and the monitor surveys. Further details regarding 4D acquisition and processing can be found in Eiken et al. (2000).

## Prestack versus poststack seismic data analysis

Most 4D studies use poststack seismic data for qualitative interpretation (Landrø and Stammeijer, 2004). For quantitative 4D analysis, prestack time-lapse data have been used to estimate pressure and saturation changes simultaneously (Tura and Lumley, 1999; Landrø, 2001; Guilbot and Smith, 2002; Røste et al., 2006). For $\mathrm{CO}_{2}$ monitoring purposes it is desirable to estimate thicknesses of the $\mathrm{CO}_{2}$ layers as well as the velocity change caused by the $\mathrm{CO}_{2}$ injected into the sand layer. The ultimate goal is to estimate the thickness of each $\mathrm{CO}_{2}$ layer. However, because measured traveltime shifts depend on thickness as well as on velocity (which depends on saturation), it is crucial to estimate both parameters from the 4D seismic data. We do not know the exact $\mathrm{CO}_{2}$ saturation, so it is hard to estimate the velocity of the sand layer containing $\mathrm{CO}_{2}$. Therefore, prestack time-lapse seismic data might offer a way to estimate velocity and thickness changes for the $\mathrm{CO}_{2}$ layers.

In this work, we study some of the more important time-lapse effects in the prestack domain to measure velocity and thickness changes within the reservoir. For prestack time-lapse seismic analysis, it is more practical to compare estimated velocity changes than the actual traveltime changes because the time shifts vary with offset (Røste et al., 2006).

The paper is organized as follows: In the first section, we outline some of the rock-physics properties of $\mathrm{CO}_{2}$ and estimate velocity changes resulting from $\mathrm{CO}_{2}$ injection using rock-physics models. These velocity estimates are compared to the changes determined from the time-lapse seismic data only. Because our technique is developed for a single-layer case, we discuss briefly how such an event was selected from the Sleipner data set. Then we derive a method combining 4D amplitude and traveltime analysis to estimate velocity and thickness changes simultaneously. This method is compared to a more conventional method combining time shifts and rock physics.

## ROCK PHYSICAL PROPERTIES OF $\mathrm{CO}_{2}$

This section investigates potential velocity changes in the Utsira Sand based on rock and fluid properties. A typical gamma-ray well $\log$ is shown in Figure 2. It clearly shows the thick Utsira Sand between 820 and 1030 m . Six or seven thin layers can be interpreted within the Utsira Sand. Apart from these layers, the sand appears to be homogeneous. The P-wave velocity is slightly above $2000 \mathrm{~m} / \mathrm{s}$ within the sand layer. A $100-\mathrm{m}$-thick high-velocity shale layer overlies the Utsira Sand. Another high-velocity shale layer approximate-
ly 10 m thick is observed 20 m deeper in the sand. Apart from these high-velocity layers, the sand velocity is nearly constant.

The $\mathrm{CO}_{2}$ bulk modulus and density variations impact velocitychange estimations and therefore are addressed. Introducing $\mathrm{CO}_{2}$ into the Utsira Formation has a dramatic effect on reflectivity and significantly interferes with other events.

## Bulk modulus of $\mathrm{CO}_{\mathbf{2}}$

Because of the relationship between the temperature and density of $\mathrm{CO}_{2}$ at reservoir pressures, it is important to know the formation temperature accurately. Uncertainty in formation temperature measurements at Sleipner introduces significant uncertainty in the estimates of $\mathrm{CO}_{2}$ bulk modulus and density, both of which are critical inputs to the Gassmann fluid substitution model. For the Utsira Formation, two measured temperatures of $27^{\circ} \mathrm{C}$ (below the top of the Utsira) and $37{ }^{\circ} \mathrm{C}$ (near the injection point) are reported. At higher temperatures, the density and bulk modulus of $\mathrm{CO}_{2}$ are lower, resulting in lower seismic velocities. However, the decrease in the seismic velocities caused by temperature variations is not very significant.
Taking the measurement uncertainties into account, we focus on calculating the bulk modulus and density of $\mathrm{CO}_{2}$ for a range of pressure and temperature conditions considered representative of the Utsira Formation. We use $\mathrm{CO}_{2}$ Therm, a software package developed by Erik Lindeberg at SINTEF Petroleum Research, based on an equation of state derived by Span and Wagner (1996). The effects of pressure and temperature are shown in Figure 3. At the temperature interval of interest, i.e., $27^{\circ}-37^{\circ} \mathrm{C}$, we note a significant change in the density and bulk modulus of $\mathrm{CO}_{2}$ : For a formation undergoing a temperature increase from $27^{\circ} \mathrm{C}$ to $37^{\circ} \mathrm{C}$ (assuming a pressure of 100 bar), the bulk modulus and density of $\mathrm{CO}_{2}$ decreases by $53 \%$ and $15 \%$, respectively.
As $\mathrm{CO}_{2}$ displaces brine in the Utsira Sand, there is a decrease in seismic velocity. A calibrated Gassmann model reflects this dependency of the P-wave velocity on $\mathrm{CO}_{2}$ saturation and temperature variation (see Figure 4 and Table 1). However, the effect of temperature on the rock velocities seems less significant than from the variations in $\mathrm{CO}_{2}$ density. As seen from Figure 4, for an increase of $10^{\circ} \mathrm{C}$, the reduction in the seismic velocities is less than $6 \%$. The variation in temperature therefore has minimal impact on seismic velocities. However, for the density changes, the temperature effect is more pronounced. For a temperature of $27^{\circ} \mathrm{C}$, the density of $\mathrm{CO}_{2}$ is close to $800 \mathrm{~kg} / \mathrm{m}^{3}$, which is not too different from that of water. A porosity of $37 \%$ yields a density change of $4 \%$. Therefore, we assume in our analysis that density changes are negligible compared to velocity changes. In cases where density changes are more significant, it is straightforward to include a density change in our equations, but then this density-change estimate needs to be based on rock-physics input.

The velocity changes in the sand are tied directly to the time shifts used for volumetric estimates. Although we may be able to estimate the volume of the $\mathrm{CO}_{2}$ layers, it might remain a challenge to estimate the total mass of the injected $\mathrm{CO}_{2}$ because of the uncertainty in density related to actual formation temperature.

According to reservoir engineering reports for the $\mathrm{CO}_{2}$ project at Sleipner, the expected pore-pressure changes are minor. This is caused by the excellent permeability and porosity of the Utsira Sand. Therefore, we neglect pressure effects in this study. Although some pressure variations close to the injection point will occur, we believe these effects are minor for the distal $\mathrm{CO}_{2}$ layer we are studying.

## Reflectivity of $\mathrm{CO}_{2}$ swept sands

Previous studies (Arts et al., 2004) have shown that the 4D signal caused by the $\mathrm{CO}_{2}$ injection into the Utsira Sand at Sleipner is strong, probably three to four times stronger than typical 4D signals from producing hydrocarbon reservoirs. As shown in Figure 5 for prestack single-offset ( 318 m ) data, the root-mean-square (rms) amplitude increase for the entire Utsira Formation is between 1.5 and 2 when the 1994 survey is compared to the 1999 survey. Arts et al. (2004) explain this significant amplitude change by a combined effect of a significant velocity drop caused by the $\mathrm{CO}_{2}$ and seismic tuning effects.

Using the calibrated Gassmann model presented in the previous section, we find that for the top Utsira interface, the fluid change corresponds to an increase in the reflection coefficient at zero offset by


Figure 3. The (a) density and (b) bulk modulus of $\mathrm{CO}_{2}$ as a function of temperature for four different pressure values, based on an equation of state by Span and Wagner (1996). Measurements show formation temperature between $27^{\circ} \mathrm{C}$ and $37^{\circ} \mathrm{C}$.
approximately four times compared to the base survey (Figure 6). This number is associated with some degree of uncertainty, mainly because the background or initial parameter values for the shale and sand vary laterally. The input parameters used to obtain the reflection

Table 1. Rock properties used in the Gassmann modeling of the P -wave response to the gradual increase of $\mathrm{CO}_{2}$. Other input parameters are $\phi=37 \%, K_{\text {grain }}=36.9 \mathrm{GPa}, K_{\text {frame }}$ $=2.56 \mathrm{GPa}, V_{\mathrm{P}_{\text {sand }}}=2050 \mathrm{~m} / \mathrm{s}, V_{\mathrm{S}_{\text {sand }}}=643 \mathrm{~m} / \mathrm{s}$, and $\rho_{\text {solid }}$ $=2650 \mathrm{~kg} / \mathrm{m}^{3}$.

| Seismic <br> parameters | $K_{\mathrm{CO}_{2}}$ <br> $(\mathrm{GPa})$ | $K_{\mathrm{H}_{2} \mathrm{O}}$ <br> $(\mathrm{GPa})$ | $\rho_{\mathrm{H}_{2} \mathrm{O}}$ <br> $\left(\mathrm{kg} / \mathrm{m}^{3}\right)$ | $\rho_{\mathrm{CO}_{2}}$ <br> $\left(\mathrm{~kg} / \mathrm{m}^{3}\right)$ |
| :--- | :---: | :---: | :---: | :---: |
| $\left(27^{\circ} \mathrm{C}, 100 \mathrm{bar}\right)$ | 0.136 | 2.28 | 1020 | 800 |
| $\left(37^{\circ} \mathrm{C}, 100 \mathrm{bar}\right)$ | 0.064 | 2.34 | 1070 | 680 |



Figure 4. P-wave velocity versus changes in water saturation, based on Gassmann modeling for a fixed pressure value of 100 bar and two different values of assumed formation temperature. The $\mathrm{CO}_{2}$ bulk modulus is $K_{\mathrm{CO} 2}$. The parameters used in the substitution model are given in Table 1.


Figure 5. Seismic rms amplitude level in Utsira Formation for the single offset at 318 m and the inline 1839 passing through the $\mathrm{CO}_{2}$ plume for a window of 400 ms . The distance between each crossline is 12.5 m . The solid line indicates the rms level in the base survey; the dotted line is rms values corresponding to the 1999 survey. An amplitude increase of about 1.9 times above the injection point and about 1.3 times for the whole inline is observed.
curves (using Smith and Gidlow's approximation from 1987) are given in Table 2. The extra amplitude increase caused by tuning effects can be up to $100 \%$ (Widess, 1973; Ostrander, 1984). A comprehensive study of the tuning effects for several of the intrasand layers is presented by Arts et al. (2004). It is also important to note that intrasand reflections (Figure 7) are not interpretable on the base survey but are stronger than the top Utsira reflection after $\mathrm{CO}_{2}$ injection. Figure 7 shows the amplitude changes observed at top Utsira and the strongest intrareservoir event (marked M1).

The first water-layer multiple of event M1 is a strong, prominent event on the 1999 poststack data. The water depth at the injection site is about 80 m , corresponding to a multiple period of 108 ms (assuming a water velocity of $1480 \mathrm{~m} / \mathrm{s}$ ). For the near-offset prestack data ( 318 m ), the amplitude ratio between the first-order multiple and event M1 is on the order of 0.6 , corresponding to a sea-bottom reflection coefficient of approximately 0.3 . Comparing this first-order multiple amplitude to the typical reflection strength on the base survey, we find that it is stronger or of the same strength as interpretable reflections from the base survey. This means that a conventional time-shift analysis (Landrø, 2001; Hatchell and Bourne, 2005; Røste et al., 2006) is not possible at this location without taking this multiple problem into account.

Ghaderi and Landrø (2005) attempted to avoid this issue by using reflections below the Utsira Sand that is not heavily influenced by multiples. However, these results might be inconclusive because of the interference between multiples and reflections below the reservoir. Although conventional demultiples have been applied to the poststack data, remaining multiple energy causes problems for accurate time-shift estimation. For the prestack data, no demultiple was used prior to analysis.

We therefore seek a thin $\mathrm{CO}_{2}$ layer that does not create a strong multiple that might interfere with the base Utsira event. The multiple problems associated with such an isolated $\mathrm{CO}_{2}$ event should be far less than for the examples we discuss.

## SELECTING AN EVENT WITH MINOR MULTIPLE INTERFERENCE PROBLEMS

As described, strong multiple energy from the injected $\mathrm{CO}_{2}$ layers situated below thin shale layers interferes with most of the reflections below the $\mathrm{CO}_{2}$ plume. However, a single, fairly weak $\mathrm{CO}_{2}$ response is found southwest of the injection point (see Figure 8). In Figure 9, the location of this event (marked A) is at about 960 ms and is highlighted in blue color. Because the amplitude change for the event is weaker than those in the central part of the $\mathrm{CO}_{2}$ plume, we assume the interference between water-layer multiples from this event and other events is likely to be less pronounced. Thus, we analyze time shifts resulting from velocity changes at event A at the reflection corresponding to the base Utsira (event B).

The subsequent development of this event during the three consecutive time-lapse acquisitions is shown in Figure 10. Figure 11 shows interleaved traces from 1994 and 1999 for event B. Notice the systematic time shift for each of the pairs. This is clearly illustrated in Figure 12, where a selected trace for all four seismic surveys is compared. The reason for this systematic time


Figure 6. Modeling the Zoeppritz reflection coefficient for various incidence angles and four different $\mathrm{CO}_{2}$ saturation values $S_{\mathrm{CO}_{2}}$. We use a standard calibrated Gassmann type of fluid substitution.

Table 2. Seismic parameters used in the numerical example (assuming $z=820 \mathrm{~m}, p=100 \mathrm{bar}$, and $T=27^{\circ} \mathrm{C}$ ). Other input parameters are $\phi=37 \%, V_{\mathrm{P}_{\text {shale }}}=2270 \mathrm{~m} / \mathrm{s}, V_{\mathrm{s}_{\text {shale }}}$ $=850 \mathrm{~m} / \mathrm{s}, \rho_{\text {shale }}=2100\left(\mathrm{~kg} / \mathrm{m}^{3}\right), \rho_{\text {solid }}=2650 \mathrm{~kg} / \mathrm{m}^{3}, \rho_{\mathrm{H}_{2} \mathrm{O}}$ $=1020\left(\mathrm{~kg} / \mathrm{m}^{3}\right)$, and $\rho_{\mathrm{CO}_{2}}=800 \mathrm{~kg} / \mathrm{m}^{3}$.

| $S_{\mathrm{CO}_{2}}$ <br> $(\%)$ | $V_{\mathrm{P}}$ <br> $(\mathrm{m} / \mathrm{s})$ | $V_{\mathrm{S}}$ <br> $(\mathrm{m} / \mathrm{s})$ | $\rho$ <br> $\left(\mathrm{kg} / \mathrm{m}^{3}\right)$ |
| :--- | :---: | :---: | :---: |
| 0 | 2050 | 643 | 2047 |
| 20 | 1568 | 645 | 2030 |
| 50 | 1470 | 649 | 2006 |
| 100 | 1437 | 656 | 1965 |



Figure 7. Two prestack seismic vintages from Utsira, illustrating effectively the presence of $\mathrm{CO}_{2}$ through increased reflectivity caused in 1999 (right) compared to preinjection case acquisition in 1994 (left). The base Utsira reflection is shifted, and traveltime picking is obscured because of possible interference with remaining sea-bottom multiples energy from event M1 marked around 1050 ms .


Figure 8. The rms amplitude map (using a 20-ms window centered about 970 ms ) of the Utsira Sand on the 1999 survey on prestack data from the single offset at 318 m . Notice the prominent $\mathrm{CO}_{2}$ sideplume event propagating to the southwest, marked by the dashed yellow circle. The three parallel yellow lines within the yellow circle represent the three crosslines where the amplitude and time-shift measurements have taken place.
shift, or pushdown, is $\mathrm{CO}_{2}$ event A , developing away from the main plume.

The amplitude pattern for event $B$ is more complicated to explain because this is the result of the signal passing through a thin layer causing subtle interference effects that influence the amplitude level at base Utsira. The method we develop in the next section is, however, independent of amplitude measurements for the base reflector and takes into account only time shifts measured on the base reflector and the amplitude measured for event A.

## THEORY - COMBINING 4D TIME SHIFT AND AMPLITUDE ANALYSIS

The attempt to estimate the velocity change caused by the presence of $\mathrm{CO}_{2}$ in the Utsira Formation by Ghaderi and Landrø (2005) takes into account the whole plume and is based only on the timeshift measurements on the base of Utsira. This method estimates the relative change in velocity to be approximately $10 \%$.

## Traveltime shift versus offset

Consider a homogeneous medium where a thin layer is introduced (see Figure A-1, Appendix A). Assuming small offset, the relative traveltime shift caused by a single-layer event is given by (see Appendix $A$, equation $A-9$ )

$$
\begin{equation*}
\frac{\Delta T}{T}=-\frac{\Delta V}{V} \frac{\Delta z}{z} \tag{2}
\end{equation*}
$$

where $T$ is the two-way traveltime to the base; $V$ and $z$ are the velocity and thickness of the homogeneous layer, respectively; and $\Delta z$ is the thickness of the $\mathrm{CO}_{2}$ layer. For the limit $\Delta z \rightarrow z$, equation 2 reproduces equation 1 . In equation 2 , the relative traveltime shift is independent of offset. This implies that it is not possible to discriminate between velocity and thickness by exploiting only the relative traveltime shifts versus offset. Therefore, we introduce 4D amplitude changes as a second equation to estimate velocity and thickness changes simultaneously.

Combining 4D amplitude and traveltime information - Amplitude variation versus time shift

Consider the same simple configuration as above where a thin layer is embedded in a homogeneous layer (Figure B-1, Appendix B). Now we consider reflection from the top and base of the layer. The resulting signal from this thin layer is

$$
\begin{equation*}
\left|S_{d}(\omega)\right|=\frac{|P(\omega)| \Delta R^{F}}{z_{1}} \sin \left(\frac{\Delta z \omega}{V_{2} \cos \theta}\right) \tag{3}
\end{equation*}
$$

Here, $P(\omega)$ is the pulse amplitude spectrum; $\Delta R^{F}$ is the change in reflectivity resulting from fluid saturation in the thin layer; $\theta$ is the incidence angle; $V_{2}$ and $\Delta z$ are the velocity and thickness of the thin layer, respectively; and $\omega$ denotes circular frequency. Using equation 2 , we see that $\Delta z / z$ $=-(\Delta T / T)\left(V_{1} / \Delta V\right)$, where $V_{1}$ is the $p$-wave ve-

Figure 9. Side-plume event and its influence on the base of Utsira. The areas of interest are event A, which is part of the main $\mathrm{CO}_{2}$ plume, and the part of the base of Utsira encircled by a dashed line, denoted as event $B$.

locity in sand. Substituting this into equation 3 and assuming $\Delta \rho / \rho \ll 1$ results in

$$
\begin{align*}
\left|S_{d}(\omega)\right|= & \frac{|P(\omega)| \omega}{2 V_{1} \cos ^{3} \theta} \frac{\Delta T}{T} \\
& \times \operatorname{sinc}\left(-\frac{\Delta T}{T} \frac{z_{1} \omega}{\Delta V \cos \theta}\right) \tag{4}
\end{align*}
$$

A well calibration to the top Utsira can be used to determine the amplitude value of the effective pulse $P$ in equation 4. Note that $P$ includes transmission and absorption losses and is not the same as the source wavelet measured in, for instance, the water layer. However, if one assumes that a proper well tie can be done to an interface close to the $\mathrm{CO}_{2}$ layer, it is possible to determine $P$.

This is an interesting result because the only unknown in equation 4 is $\Delta V$. Combining equation 4 with equation 2, we can determine the thickness $\Delta z$. This means that it is possible to estimate the $\mathrm{CO}_{2}$ thickness, given that the reflection coefficient of the top of the $\mathrm{CO}_{2}$ anomaly can be estimated from the seismic amplitude and that the traveltime shift caused by this anomaly is measured on an interface below the anomaly.

## FIELD DATA EXAMPLE

Here, we test the methodology using prestack seismic data from the selected areas of the seismic vintages shown in Figure 10. In addition, we also test the direct method using time shifts and rock physics. In Figure 10, a somewhat weaker seismic event (around 1070 ms ) occurs below target event A (which is around 960 ms in Figure 9 ) in surveys from 2001 and 2002. The traveltime difference between these two events and the amplitude ratio indicate this is a multiple. However, we cannot rule out the possibility that this event is a second $\mathrm{CO}_{2}$ layer. Despite this, we assume that the influence of this event is negligible for our analysis, mainly based on the much weaker amplitude for this event.
tainties involved in the estimate for the zero-offset reflection coefficient as well as the average amplitude strength measured at top Utsira. Despite these uncertainties, we use this value for all surveys. This means that by choosing a constant value for $P$, the relative changes in our estimated thickness and velocity changes should suffer less from this uncertainty, but there could still be an absolute error that is unaccounted for. Picking near-offset amplitudes over a large

Crossline number


Figure 10. Side-plume event during the three time-lapse 3D seismic surveys. The distance between each crossline is 12.5 m .

## Estimating the source amplitude

To estimate the pulse amplitude of the seismic source wavelet $P$, we use the top of the Utsira Formation for calibration. From well logs, we estimate a zero-offset reflection coefficient of -0.063 . The average measured seismic amplitude for this event is $S_{d}=0.36$. The depth of top Utsira is approximately 820 m and average two-way traveltime is 890 ms , yielding an approximate estimate for the pulse strength (using equation B-1) of

$$
\begin{equation*}
P=\frac{2 z S_{d}}{T^{2} R_{0}} \approx \frac{2 \times 820 \times 0.46}{(0.89)^{2} 0.063}=11,943 \tag{5}
\end{equation*}
$$

It is important to note that this simple procedure involves significant uncertainties in the pulse estimate because there are large uncer-


Figure 11. Constant-offset ( 318 m ) traces at event B, with interleaved traces from 1994 (left) and 1999 (right). Notice the systematic shift for each pair of traces.


Figure 12. Effect of the $\mathrm{CO}_{2}$ plume on the base of the Utsira Sand on a selected trace, corresponding to (inline, crossline) $=(1821,1060)$ at event B. Note the systematic increase in the time shifts after each survey. Also, the amplitude effects are quite prominent.


Figure 13. Amplitude variation versus the relative traveltime shift. The single points correspond to the actual measured amplitude data from each of the time-lapse vintages in 1999, 2001, and 2002. Each point corresponds to an average of measurement values from three crosslines, where the bars indicate the standard deviation and the shaded area indicates the spread of data. The curves correspond to the theoretical model based on equation 4 . Each curve is labeled with the value of the velocity change.

Table 3. Variation in the thickness of the $\mathbf{C O}_{2}$ layer according to the method of combining time shifts and 4D amplitudes.

| Survey year | $\Delta V(\mathrm{~m} / \mathrm{s})$ | $\Delta z(\mathrm{~m})$ |
| :--- | :---: | :---: |
| 1999 | 200 | 15 |
| 2001 | 400 | 15 |
| 2002 | 500 | 15 |

area ( 4000 common depth points) gave a standard deviation for $S_{d}$ of 0.029 or $8 \%$. The uncertainty in the reflection coefficient is probably larger because variation in cap-rock properties and inaccuracies introduced by upscaling from well logs introduce systematic errors in the estimated reflection coefficient.

## Estimation of velocity and thickness changes using 4D time shifts and amplitude changes

From equation 4, we observe that there might be several values of the unknown $\Delta V$ parameter giving the same observed $S_{d}$ value. For simplicity, we introduce a simple graphical solution procedure as shown in Figure 13. The measured 4D seismic data are shown with their estimated standard deviations (one standard deviation) for both of the measured parameters (relative time shifts along the horizontal axis and 4D amplitude changes along the vertical axis). The combined effect of the two standard deviations gives rise to the shaded ellipses in the figure, and our strategy is to choose solutions from the modeled curves that are as close as possible to the center of the three ellipses.

For the modeled curves in Figure 13, the following parameters were used: $f=50 \mathrm{~Hz}, \theta=9.6^{\circ}, V_{1}=1850 \mathrm{~m} / \mathrm{s}$, and $z_{1}=943 \mathrm{~m}$. For the overburden velocity $V_{1}$, an average value has been used, and the depth of the overburden is taken from the well log.

From an estimate of the velocity change $\Delta V$, the variation in thickness is calculated as (equation 2)

$$
\begin{equation*}
\Delta z=-z_{1} \frac{V_{1}}{\Delta V} \frac{\Delta T}{T} \tag{6}
\end{equation*}
$$

The results are given in Table 3.

## Estimation of velocity and thickness changes using traveltime shifts and rock physics

If we assume that the average velocity within the $\mathrm{CO}_{2}$ layer and the corresponding relative velocity change (caused by the $\mathrm{CO}_{2}$ injection) are known from rock-physics measurements or a calibrated Gassmann model, we can exploit this information to estimate the thickness of a thin $\mathrm{CO}_{2}$ layer within a homogeneous background model.

In this case, we assume that the $\mathrm{CO}_{2}$ is trapped below a thin shale layer and that the shale layer is thin compared to the thickness of the $\mathrm{CO}_{2}$ layer. Typically, we assume that the shale layer is only $1-2 \mathrm{~m}$ thick and that the $\mathrm{CO}_{2}$ layer is around $10-20 \mathrm{~m}$. If the relative velocity change is known, we can use equation 2 directly to estimate the thickness change. A slightly more accurate method for the zero-offset case can be derived as follows:

$$
\begin{equation*}
\Delta T_{0}=\frac{2 \Delta z}{V_{2}}-\frac{2 \Delta z}{V_{1}} \tag{7}
\end{equation*}
$$

where $V_{1}$ and $V_{2}$ are the velocities before and after $\mathrm{CO}_{2}$ injection, respectively, and $\Delta z$ is the thickness of the $\mathrm{CO}_{2}$ layer. If we assume that
$\Delta T_{0}$ can be measured at an interface below the $\mathrm{CO}_{2}$ layer, the thickness of the $\mathrm{CO}_{2}$ layer is

$$
\begin{equation*}
\Delta z=\frac{V_{1} V_{2}}{2\left(V_{1}-V_{2}\right)} \Delta T_{0} \tag{8}
\end{equation*}
$$

As discussed previously, for the Utsira Formation at Sleipner, typical values for velocities from well-log analysis for $100 \%$ water-saturated sand at $2050 \mathrm{~m} / \mathrm{s}$ and assuming that the end state is fully $\mathrm{CO}_{2}$ saturated, we obtain a velocity of $1440 \mathrm{~m} / \mathrm{s}$. Using these values in equation 8 yields

$$
\begin{equation*}
\Delta z \approx 2400 \Delta T_{0} \tag{9}
\end{equation*}
$$

assuming that the time shift is estimated in seconds. In practice, however, we do not know the $\mathrm{CO}_{2}$ saturation; hence, the velocity change between monitor and base surveys might be associated with significant uncertainties. Figure 14 shows estimated zero-offset (we assume 318 m is close to zero offset) traveltime shifts for a selected inline at Utsira. Using equation 9 , we estimate the $\mathrm{CO}_{2}$ thicknesses. The results are given in Table 4.

The major difference between the two methods is that the method combining amplitude and traveltime shifts gives a gradual decrease in velocity over calendar time; the rock-physics-based method predicts constant velocity change. Hence, the first method predicts a more-or-less constant thickness of the $\mathrm{CO}_{2}$ layer, whereas the second method predicts a constant velocity and a gradual increase in the thickness of the $\mathrm{CO}_{2}$ layer.

A possible explanation for the lower velocity-change value in 1999 compared to the Gassmann prediction is that the saturation distribution is more patchy in 1999 than in 2001 and 2002. By patchy distribution, we mean there might be one or two thin shale layers within the thickness of 15 m that trap individual $\mathrm{CO}_{2}$ layers with a thickness below seismic resolution. Changes in amplitudes because of saturation differences within Utsira are addressed by Carcione et al. (2006). They relate varying saturation to the presence of thin, isolated shale layers. They find that the P-wave velocity discriminates between uniform and patchy saturations for brine saturations above $60 \%$. They support their conclusion by numerical modeling, taking into account patchy saturation and attenuation.

## DISCUSSION

A major objective of this work is to develop a quantitative 4D analysis method to estimate $\mathrm{CO}_{2}$ velocity and layer-thickness changes. These estimates can be used to estimate the volume of injected $\mathrm{CO}_{2}$ into a saline aquifer. To illustrate this, we have used timelapse seismic data from 1999, 2001, and 2002 acquired at the Sleipner field. To simplify the analysis, we selected a case where the $\mathrm{CO}_{2}$-saturated layer is isolated from the main plume. The results obtained for the $\mathrm{CO}_{2}$ thickness are 15 m for all three vintages, leading to expectation that the velocity is also constant. However, the estimated velocity changes are 200,400 , and $500 \mathrm{~m} / \mathrm{s}$ for the three vintages. One possible way to interpret this result is to introduce patchy saturation changes within the 15 -m-thick $\mathrm{CO}_{2}$ layer. On the other hand, fixing the velocity change to $30 \%$ reduction leads to a gradual increase in the $\mathrm{CO}_{2}$ layer thickness.

It is not straightforward to implement our method of combining time-shift and amplitude analysis for multiple layers of $\mathrm{CO}_{2}$ because the time shift measured for an interface below the sand layer will provide information only about the average time shift for all $\mathrm{CO}_{2}$ layers stacked above each other. A way to circumvent this problem might be to use a single-layer plume, as described here, to determine the velocity change and then to assume that the velocity change is more or less constant for the multistack case. However, this assumption means that the $\mathrm{CO}_{2}$ saturation for all layers must be approximately equal, and that is unlikely.

Therefore, other methods must be developed to handle this case. One approach might be to consider amplitude changes versus offset, similar to a procedure used by Landrø (2001) to separate pressure and saturation changes in the time-lapse data. But because we have to assume thin layers, the amplitude changes with offset might be harder to resolve for independent estimates of velocity and thickness changes. Another approach to solve the multilayer $\mathrm{CO}_{2}$ problem is to combine reservoir fluid-flow simulation with seismic 4D modeling and invert for velocity and thickness changes. However, such an approach requires knowledge of vertical and horizontal permeability distributions within the heterogeneous sand layer.

Because the equation derived for estimating the velocity change from time-shift and amplitude changes has multiple solutions (inverse of a sinc function), we propose a graphical technique to visualize various solutions and compare to the observed 4D data. The spread in the estimated time shifts is significant, and this leads to significant uncertainties in the estimated velocity and thickness changes.


Figure 14. Estimated time shift caused by the $\mathrm{CO}_{2}$ side-plume event, measured on the base of the Utsira Sand.

Table 4. Thickness variation of the $\mathrm{CO}_{2}$ layer according to rock physics and time shifts from seismic data and equation 9.

| Thickness <br> variation (m) | 1999 | 2001 | 2002 |
| :--- | :---: | :---: | :---: |
| $\Delta z$ | 4 | 8 | 10 |

## CONCLUSIONS

Injection of $\mathrm{CO}_{2}$ into a sand layer causes significant amplitude and traveltime changes on time-lapse seismic data. Based on field data from the Sleipner $\mathrm{CO}_{2}$ storage project, we find that it is possible to discriminate between velocity and thickness changes caused by $\mathrm{CO}_{2}$ injection. Thin $\mathrm{CO}_{2}$ layers are formed below even thinner shale layers within the sand body where the $\mathrm{CO}_{2}$ is injected.

All analysis is done on a near-offset narrow stack (offset range 243-393 m). We estimate a velocity change of around $200 \mathrm{~m} / \mathrm{s}$ three years after injection start and $500 \mathrm{~m} / \mathrm{s}$ two to three years later.

A combination of 4D amplitude and time-shift analysis enables us to discriminate between velocity and thickness changes within thin $\mathrm{CO}_{2}$ layers. The 4D amplitude analysis is based on simple tuning equations. Our method is limited to only one $\mathrm{CO}_{2}$ layer within the sand body, and the method is tested successfully for such a singlelayer event. Most of the $\mathrm{CO}_{2}$ layers caused by the injection, however, are stacked on top of each other as separated layers (typically 20-50 m between each layer).

When 4D time shifts and 4D amplitude changes are combined to estimate velocity and thickness changes simultaneously, we find that the thickness for a single $\mathrm{CO}_{2}$ layer at Utsira remains constant from 1999 to $2002(15 \mathrm{~m})$; the velocity gradually changes from $200 \mathrm{~m} / \mathrm{s}$ in 1999 to $400 \mathrm{~m} / \mathrm{s}$ in 2001 and $500 \mathrm{~m} / \mathrm{s}$ in 2002. If we fix the velocity change by using rock physics (Gassmann) and assume a constant $\mathrm{CO}_{2}$ saturation for the entire layer, we find that the thickness of the $\mathrm{CO}_{2}$ layer increases from 4 m in 1999 to 8 m in 2001 and to 10 m in 2002. These results are not too far from each other because a lower velocity change will mean several thin, separated $\mathrm{CO}_{2}$ layers in a patchy saturation case, leading to an effective thickness less than 15 m .

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## APPENDIX A

## TIME SHIFT VERSUS OFFSET

When $\mathrm{CO}_{2}$ is injected into an aquifer formation, new thin $\mathrm{CO}_{2}$ layers might appear below thin shale layers (typically $1-3 \mathrm{~m}$ thick). Because of the strong velocity decrease caused by the $\mathrm{CO}_{2}$ injection, these layers are observed as strong events.

Figure $\mathrm{A}-1$ shows a case where one such $\mathrm{CO}_{2}$ layer is created within a homogeneous sand layer. Consider the sketch on the left in Figure A-1. This represents a homogeneous aquifer sand body. The sketch on the right represents the situation when a thin layer of $\mathrm{CO}_{2}$ is introduced. From the figure, we note that $z=z_{1}+\Delta z+z_{2}, \theta$ is the angle in the nonperturbed media prior to $\mathrm{CO}_{2}$ injection, and $\theta_{1}$ and $\theta_{2}$ are incidence angles for the perturbed media caused by $\mathrm{CO}_{2}$, shown in Figure A-1.

The two-way traveltimes in the unperturbed and perturbed media are given as

$$
\begin{gather*}
T_{1}=\frac{2 z}{V \cos \theta},  \tag{A-1}\\
T_{2}=\frac{2 z}{V \cos \theta}\left[\left(1-\frac{\Delta z}{z}\right) \frac{\cos \theta}{\cos \theta_{1}}+\frac{\Delta z}{z\left(1+\frac{\Delta V}{V}\right)} \frac{\cos \theta}{\cos \theta_{2}}\right] . \tag{A-2}
\end{gather*}
$$



Figure A-1. Base (left) and monitor (right) cases, representing pre- and post- $\mathrm{CO}_{2}$ injection. The objects are not to scale and are exaggerated for clarity. It is assumed that the shale layer is much thinner than the $\mathrm{CO}_{2}$ layer and its effect on traveltime is not taken into account in calculations but is included in the figure above the $\mathrm{CO}_{2}$ layer for illustration.

The relative time shift is then given as

$$
\begin{align*}
\frac{\Delta T}{T_{1}}= & \left(1-\frac{\Delta z}{z}\right) \sqrt{\frac{1+\tan ^{2} \theta_{1}}{1+\tan ^{2} \theta}} \\
& +\frac{\frac{\Delta z}{z}}{\left(1+\frac{\Delta V}{V}\right)} \sqrt{\frac{1+\tan ^{2} \theta_{2}}{1+\tan ^{2} \theta}}-1 \tag{A-3}
\end{align*}
$$

The fixed-offset condition means that $\left(x_{1}=x_{2}\right)$

$$
\begin{equation*}
\tan \theta=\left(1-\frac{\Delta z}{z}\right) \tan \theta_{1}+\frac{\Delta z}{z} \tan \theta_{2} \tag{A-4}
\end{equation*}
$$

We also recall Snell's law:

$$
\begin{equation*}
\sin \theta_{2}=\left(1+\frac{\Delta V}{V}\right) \sin \theta_{1} \tag{A-5}
\end{equation*}
$$

By combining equations $\mathrm{A}-4$ and $\mathrm{A}-5$, it is possible to show that

$$
\begin{equation*}
\tan \theta_{2} \approx \tan \theta_{1}\left(1+\frac{\Delta V}{V \cos ^{2} \theta_{1}}\right) \tag{A-6}
\end{equation*}
$$

and

$$
\begin{equation*}
\tan \theta \approx \tan \theta_{1}\left(1+\frac{\Delta V \Delta z}{V z \cos ^{2} \theta_{1}}\right) \tag{A-7}
\end{equation*}
$$

Inserting equations A-6 and A-7 into equation A-3 and using the Taylor expansion to the third order yields

$$
\begin{align*}
\frac{\Delta T}{T}= & -\frac{\Delta V}{V} \frac{\Delta z}{z}\left[1+\frac{\Delta V}{V}\left(\tan ^{2} \theta-1-\frac{1}{2} \tan ^{4} \theta\right.\right. \\
& \left.\left.-\frac{1}{2} \frac{\tan ^{2} \theta}{\cos ^{2} \theta}\right)\right] \tag{A-8}
\end{align*}
$$

A simpler approximation that can be derived from equations A-6 and A-7 is

$$
\begin{equation*}
\frac{\Delta T}{T}=-\frac{\Delta V}{V} \frac{\Delta z}{z}\left(1-\frac{\Delta V}{V} \cos ^{2} \theta\right) \tag{A-9}
\end{equation*}
$$

For larger values of $\Delta V / V$, it is recommended to keep the term in the denominator, that is,

$$
\begin{equation*}
\frac{\Delta T}{T}=-\frac{\Delta V}{V} \frac{\Delta z}{z\left(1+\frac{\Delta V}{V} \cos ^{2} \theta\right)} \tag{A-10}
\end{equation*}
$$

## APPENDIX B

## AMPLITUDE RESPONSE OF A THIN $\mathrm{CO}_{2}$ LAYER

Consider a $\mathrm{CO}_{2}$ layer of thickness $\Delta z$, embedded in a homogeneous layer with a background velocity $V_{1}$. Furthermore, assume that the depth of the $\mathrm{CO}_{2}$ layer is $z_{1}$ and the velocity of the layer is $V_{2}$ (see Figure B-1). For such a model, if the reflection coefficient of the top of the $\mathrm{CO}_{2}$ layer is $R_{0}$, then the reflection coefficient for the bottom of the thin layer is $-R_{0}$. Hence, the signals reflected from the top and the base of the thin $\mathrm{CO}_{2}$ layer are given as

$$
\begin{align*}
& S_{1}(t)=\frac{1}{2 z_{1}} P\left(t-\frac{2 z_{1}}{V_{1}}\right) R_{0} \\
& S_{2}(t)=\frac{1}{2\left(z_{1}+\Delta z\right)} P\left(t-\frac{2 z_{1}}{V_{1}}-\frac{2 \Delta z}{V_{2}}\right)\left(-R_{0}\right) \tag{B-1}
\end{align*}
$$

Here, $P(t)$ denotes the seismic source wavelet. Assuming that $\Delta z \ll z_{1}$, the amplitude spectrum of the resultant signal reflected from this layer is given as

$$
\begin{equation*}
\left|S_{d}(\omega)\right|=\left(\frac{R_{0}}{z_{1}}\right)|P(\omega)| \sin \left(\frac{\Delta z \omega}{V_{2}}\right) \tag{B-2}
\end{equation*}
$$

The traveltime change caused by the presence of $\mathrm{CO}_{2}$ (assuming zero offset and that $t_{1}=2 z_{1} / V_{1}+2 \Delta z / V_{1}$ and $t_{2}=2 z_{1} / V_{1}$ $+2 \Delta z / V_{2}$ ) is

$$
\begin{equation*}
\Delta t=-\frac{2 \Delta z}{V_{2}} \frac{\Delta V}{V_{1}} \tag{B-3}
\end{equation*}
$$

Now, still assuming a zero-offset reflection $\rho_{0} \approx \rho_{1}, \Delta V \ll V_{1}$, and using equation $\mathrm{B}-3$, the reflection coefficient at zero offset can be approximated by

$$
\begin{equation*}
R_{0} \approx-\Delta t \frac{V_{2}}{4 \Delta z} \tag{B-4}
\end{equation*}
$$

Substituting equation B-4 into equation B-2 and solving for $\Delta z$, we obtain an explicit expression for the thin-layer thickness, given as

$$
\begin{equation*}
\Delta z=\frac{V_{2}}{\omega} \operatorname{sinc}^{-1}\left(-\frac{4 S_{d}(\omega) z_{1}}{\Delta t \omega P(\omega)}\right) \tag{B-5}
\end{equation*}
$$

where $\operatorname{sinc}^{-1}$ is inverse of the sinc function, $\operatorname{sinc}(x)=\sin x / x$.
So far, we have been looking into vertical incident signals to the thin bed. For nonzero incident angles $\theta$, the thickness $\Delta z$ of the $\mathrm{CO}_{2}$ layer is expressed as $\Delta z / \cos \theta$. This results in generalization of equation B-2 to

$$
\begin{equation*}
\left|S_{d}(\omega)\right|=\left(\frac{R_{0}}{z_{1}}\right)|P(\omega)| \sin \left(\frac{\Delta z \omega}{V_{2} \cos \theta}\right) \tag{B-6}
\end{equation*}
$$

The change in reflectivity caused by fluid saturation change in the thin layer for nonzero incidence angle is (Landrø, 2001)

$$
\begin{equation*}
\Delta R^{F} \approx \frac{1}{2}\left(\frac{\Delta \rho}{\rho}+\frac{\Delta V}{V_{1}}\right)+\frac{\Delta V}{2 V_{1}} \tan ^{2} \theta \tag{B-7}
\end{equation*}
$$

Inserting equation B-7 into B-6 results in

$$
\begin{align*}
\left|S_{d}(\omega)\right|= & \frac{|P(\omega)|}{2 z_{1}}\left(\left[\frac{\Delta \rho}{\rho}+\frac{\Delta V}{V_{1}}\right]\right. \\
& \left.+\frac{\Delta V}{V_{1}} \tan ^{2} \theta\right) \sin \left(\frac{\Delta z \omega}{V_{2} \cos \theta}\right) \tag{B-8}
\end{align*}
$$

From equation 2, we see that $\Delta z / z=-(\Delta T / T)\left(V_{1} / \Delta V\right)$. Accordingly, the sinus argument in equation B-8 could be expressed as $-(\Delta T / T)\left(z_{1} \omega / \Delta V \cos \theta\right)$. Substitution in equation B-8, rearranging and assuming that $\Delta \rho / \rho \ll 1$, gives


Figure B-1. Schematics of reflection from a thin layer.

$$
\begin{equation*}
\left|S_{d}(\omega)\right|=\frac{|P(\omega)| \omega}{2 V_{1} \cos ^{3} \theta} \frac{\Delta T}{T} \operatorname{sinc}\left(-\frac{\Delta T}{T} \frac{z_{1} \omega}{\Delta V \cos \theta}\right) . \tag{B-9}
\end{equation*}
$$

Solving for $\Delta V$ gives

$$
\begin{equation*}
\Delta V=-\frac{z_{1} \omega \frac{\Delta T}{T}}{\cos \theta \times \operatorname{sinc}^{-1}\left[\frac{2 S_{d}(\omega) V_{1} \cos ^{3} \theta}{\omega P(\omega)}\left(\frac{\Delta T}{T}\right)^{-1}\right]} \tag{B-10}
\end{equation*}
$$

The result in equation B-10 combined with equation 2 determines the tuning thickness $\Delta z$.

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    ${ }^{1}$ Norwegian University of Science and Technology, Department of Petroleum Engineering and Applied Geophysics, Trondheim, Norway. E-mail: ghaderi@ ntnu.no; mlan@ipt.ntnu.no.
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